Assignment 5

443 Statistical Physics 2008/2009

Lecturer: Stefan Sint

(due Friday, 30 January 2009 during class)

Problem 1 (10/20 points)

Consider a system with N particles and Hamilton function

$$\mathcal{H}(p,q) = \sum_i \frac{1}{2m}\vec{p}_i^{\,2} + \sum_{i<j} \varphi(|\vec{q}_i - \vec{q}_j|). \tag{1}$$

Let

$$\vec{L}(p,q) = \sum_i \vec{q}_i \times \vec{p}_i, \qquad \vec{P}(p,q) = \sum_i \vec{p}_i, \tag{2}$$

be the total angular momentum and momentum respectively. Show that the phase space distribution function

$$f(p,q) = C \exp\left\{-\beta\mathcal{H}(p,q) - \vec{\gamma}\cdot\vec{L}(p,q) - \vec{\epsilon}\cdot\vec{P}(p,q)\right\}, \tag{3}$$

is time independent (here, C is a suitable normalisation constant, and $\vec{\gamma}, \vec{\epsilon}$ are constant vectors).

Problem 2 (10/20 points)

Consider a gas of N hard spheres of radius r_0 and mass m. The interaction potential between particles i and j is thus given by

$$\varphi(r_{ij}) = \begin{cases} 0 & \text{if } r_{ij} > r_0, \\ \infty & \text{otherwise,} \end{cases} \qquad r_{ij} \equiv |\vec{q}_i - \vec{q}_j|. \tag{4}$$

a) Compute the cluster integral b_2 in the large volume limit and determine the equation of state including the first correction term.

b) Compare this result with the van-der-Waals equation of state in the limit of small particle density, and determine the coefficients a and b.