Assignment 3

443 Statistical Physics 2008/2009

Lecturer: Stefan Sint

(due Tuesday, 2 December 2008 during class)

Problem 1 (6/20 points)

a) Show that a differentiable convex function f of a single variable has a positive second derivative.

c) In class we have seen the explicit expressions of the thermodynamic potentials for the ideal gas. Check the convexity/concavity properties with respect to all variables for the free energy, the internal energy of an ideal gas, when taken as functions of their natural variables. Do the same for the entropy $S = S(U, V, N)$.

Problem 2 (8/20 points)

Black body radiation

a) Show the relation

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

b) From electrodynamics we know that the radiation pressure of an isotropic field in a resonator is proportional to the energy density

$$P = \frac{1}{3}u(T),$$

where $u(T) = U/V$ is a function of temperature alone. Use part a) to establish the relation

$$u = \frac{1}{3}Tu' - \frac{1}{3}u.$$

c) Solve this equation to obtain $u(T)$.

Problem 3 (6/20 points)

Consider a system with the Gibbs' free energy

$$G(P, T, N) = RT \ln \left(\frac{aP}{(RT)^{5/2}} \right),$$

with real constants a and R. Compute the heat capacity C_P and the internal energy, $U = U(V, T, N)$ and $U = U(V, S, N)$.