1S1 Tutorial Sheet 9[1]

14-16 December 2011

Useful facts:

- The **anti-derivative** and the **indefinite integral**: For a function $f(x)$ the function $F(x)$ is the *anti-derivative* if
$$\frac{dF(x)}{dx} = f(x) \tag{1}$$
The *indefinite integral* is the family of all anti-derivatives
$$\int f(x)dx = F(x) + C \tag{2}$$
where C is *the arbitrary constant of integration.*

- **Integration table** (recall $\sec x = 1/\cos x$ and $\operatorname{cosec}(x) = 1/\sin x$)

$f(x)$	$\int f(x)dx$
$x^r\ (r \neq -1)$	$\frac{x^{r+1}}{r+1} + C$
$\sin x$	$-\cos x + C$
$\cos x$	$\sin x + C$
$\sec^2 x$	$\tan x + C$
$\sec x \tan x$	$\sec x + C$
$\operatorname{cosec}^2 x$	$-\cot x + C$
$\operatorname{cosec} x \cot x$	$-\operatorname{cosec} x + C$

- **Linearity**: if
$$\int f(x)dx = F(x) + C \qquad \int g(x)dx = G(x) + C \tag{3}$$
then
$$\int \lambda f(x)dx = \lambda F(x) + C \qquad \int [f(x) + g(x)]dx = F(x) + G(x) + C \tag{4}$$
where λ is a constant.

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- u**-substitution**: If $F'(x) = f(x)$ then the chain rule from the point of view of antiderivatives can be written in the form:

$$\int f(g(x))g'(x)dx = F(g(x)) + C \tag{5}$$

To compute an integral like the LHS one substitutes $u = g(x)$ and $du = \frac{du}{dx}dx = g'(x)dx$ and writes

$$\int f(g(x))g'(x)dx = \int f(u)du = F(u) + C = F(g(x)) + C \tag{6}$$

where in the last step one substitutes back $u = g(x)$ so that x is the independent variable.

Questions

1. (2) Find the indefinite integral of $x^3 - 4x$, $\sqrt{x} + 1/\sqrt{x}$, $x^3 - \sqrt[3]{x}$ and $x(1 + x^4)$.

2. (2) Evaluate

$$\int \frac{1}{1 - \sin x} dx \tag{7}$$

Hint: first multiply by $1 + \sin x$ above and below the line, then use the identity $\cos^2 x + \sin^2 x = 1$.

3. (2) Integrate $\tan^2 x$ and $\cot^2 x$.

Hint: start by using the identity $\cos^2 x + \sin^2 x = 1$ to rewrite the integrand.

4. (2) Solve the following integrals using u-substitution:

$$\int (x^2 - 1)^{1/3} x\, dx, \qquad \int \sin^2(x)\cos(x)\, dx \tag{8}$$

Hint: use substitutions $u = x^2 - 1$ and $u = \sin(x)$, respectively

Extra Questions

The questions are extra; you don't need to do them in the tutorial class.

1. Integrate x^4, $1/x^4$, $\sqrt[4]{x}$ and $1/\sqrt[4]{x}$.

2. Find y when

$$\frac{dy}{dx} = \sec^2 x - \sin x \tag{9}$$

with $y(\pi/4) = 1$.

3. Use the identities $\cos 2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$ to integrate $\sin^2(x/2)$ and $\cos^2(x/2)$.

4. Suppose a point moves along a curve $y = f(x)$ in the xy-plane in such a way that at each point (x, y) on the curve the tangent line has slope $-\sin x$. Find an equation for the curve, given that it passes though the point $(0, 2)$.