MA1S11 Tutorial Sheet 8[1]

7-9 December 2011

Useful facts:

- **Relative extrema**: A point $x = a$ is an relative or local maximum if there is an open interval I containing a such that $f(a) \geq f(x)$ for all x in I, a point $x = a$ is an relative or local minimum if there is an open interval I containing a such that $f(a) \leq f(x)$ for all x in I.

- **Absolute extrema**: A point $x = a$ is an absolute or global maximum if $f(a) \geq f(x)$ for all x in the domain, it is an absolute or global minimum if $f(a) \leq f(x)$ for all x in the domain. If the domain is a closed interval then one needs to compare the values of f at the relative extrema with the values of f at the endpoints of the interval to find out which are absolute or global extrema.

- **Critical points, second derivative test, concavity up/down**: If $f'(x) > 0$ a function f is **increasing** at x, if it is negative it is **decreasing**, if $f'(x) = 0$ then x is a critical point. If $f''(x) > 0$ at a critical point it is a **relative minimum**, if $f''(x) < 0$ at a critical point it is a **relative maximum**, if $f''(x) = 0$ it is undecided, it could be a **point of inflection**, a point joining a **concave up** $f''(x) > 0$ interval from a **concave down**, $f''(x) < 0$, interval.

- **Newton's method**: To find a zero of f, i.e. the solution of $f(x) = 0$ start with an initial guess x_1 and iterate using

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{1}$$

  until it converges or is obviously not working.

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Questions

1. (4) Consider the function

$$f(x) = -\frac{1}{4}x^4 + \frac{1}{3}x^3 + x^2 + 1 \tag{2}$$

   - Determine the relative and the absolute extrema of f on the closed interval $[-2, 3]$.
   - Determine the inflection points and the regions of concavity up or down.

2. (4) A person jumps from a height of $10m$ into a swimming pool. The position (in metres above water) as a function of time (in seconds) is given by

$$s(t) = 10m - \frac{1}{2}gt^2, \tag{3}$$

   where $g = 9.81m/s^2$ (metres per seconds squared) is the acceleration on earth due to the gravitational force.

   Calculate

   - How long does the person fall?
   - What is the average velocity of the fall (total length divided by duration of the fall)?
   - What is the velocity as a function of t (you need to calculate the derivative of $s(t)$ with respect to time)?
   - What is the velocity at the end of the fall? How much is this in kilometres per hour?

Extra Question

The question is extra; you don't need to do it in the tutorial class.

1. Use Newton's Method to find the root of $x^3 - x + 2$ to three decimal places using -1 as an initial guess.