23-25 November 2011

Useful facts:

- Constant: If f = c a constant f' = 0.
- **x to the r**: If $f(x) = x^r$ then $f'(x) = rx^{r-1}$.
- Trigonometric functions

$$\frac{d}{dx}\cos x = -\sin x$$
$$\frac{d}{dx}\sin x = \cos x \tag{1}$$

• Product rule:

$$\frac{d}{dx}(fg) = f\frac{dg}{dx} + \frac{df}{dx}g$$
(2)

• Quotient rule:

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{\frac{df}{dx}g - f\frac{dg}{dx}}{g^2} \tag{3}$$

• Chain rule: With u = g(x)

$$\frac{d}{dx}f(g(x)) = \frac{df}{du}\frac{du}{dx} = f'(u)g'(x).$$
(4)

• Implicit differentiation: Example: find dy/dx for $y^2 + x^2 = 1$, this gives a relationship between x and y without expressing y explicitly as a function of x. Differentiate across

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}1 = 0$$
(5)

 \mathbf{SO}

$$2x\frac{dx}{dx} + \frac{dy^2}{dx} = 0\tag{6}$$

and hence

$$2x + 2y\frac{dy}{dx} = 0\tag{7}$$

or, solving for dy/dx

$$\frac{dy}{dx} = -\frac{x}{y} \tag{8}$$

¹Stefan Sint, sint@maths.tcd.ie, see also http://www.maths.tcd.ie/~sint/1S11.html

Questions

The numbers in brackets give the numbers of marks available for the question.

1. (4) Differentiate the following functions with respect to x

$$x^3 \cos 3x, \qquad \sin(\cos x), \qquad \frac{\sin x}{\cos^2 x}, \qquad -x^2 \cos \sqrt{x}$$
(9)

2. (2) Find dy/dx for

$$3yx^6 - 4x^2 + 6\sin y^4 = 0 \tag{10}$$

3. (2) Find the slope, that is dy/dx, of the curve

$$3y^2 - 2x^2 = xy \tag{11}$$

at the point (1, 1).

Extra Questions

The questions are extra; you don't need to do them in the tutorial class.

1. Differentiate

$$\frac{1}{\sqrt{1+x}}, \quad \sin(\cos x), \quad \sin\frac{1}{1+x^2}, \quad \sqrt{\sin x}, \quad \tan 2x = \frac{\sin(2x)}{\cos(2x)}$$
(12)

2. Find dy/dx and d^2y/dx^2 for

$$\sin xy = 0 \tag{13}$$