MA1S11 Calculus, Tutorial Sheet 3[1]

26-28 October 2011

Useful facts:

- **Limit**: (Informal definition). If the value of $f(x)$ can be made as close as we like to L by taking values of x sufficiently close to a but not equal to a then we write
$$\lim_{x \to a} f(x) = L \tag{1}$$
- **One-sided Limit**: (Informal definition). If the value of $f(x)$ can be made as close as we like to L by taking values of x sufficiently close to a and greater than a then we write
$$\lim_{x \to a+} f(x) = L \tag{2}$$
If the value of $f(x)$ can be made as close as we like to L by taking values of x sufficiently close to a and less than a then we write
$$\lim_{x \to a-} f(x) = L \tag{3}$$

Questions

The numbers in brackets give the numbers of marks available for the question.

1. (2) Evaluate the limits
$$\lim_{x \to 3} \frac{(x+2)(x-3)}{x-3}, \qquad \lim_{x \to 1} \frac{x^2+4x-5}{x-1}, \qquad \lim_{x \to 2} \frac{x^2+4x-5}{x-1}. \tag{4}$$
2. (3) Consider the function $f(x) = 2(x+2)$. Find, for each of the given values $\varepsilon > 0$, a $\delta > 0$ such that for $|x-1| < \delta$ one has $|f(x) - 6| < \varepsilon$.
   a) $\varepsilon = 0.1$
   b) $\varepsilon = 0.01$
   c) $\varepsilon = 0.001$
3. (3) Consider the function $f(x) = x^2 + 2$. Determine the secant lines which go through the point $P(1,3)$ and any other point $Q(x_0, f(x_0))$ of the graph. Approximate the slope of the tangent line to the curve which passes through $P(1,3)$, by calculating the slope of the secant lines for points $Q(x_0, f(x_0))$ approaching P (consider 3-4 numerical values). What is the equation for the tangent line in $P(1,3)$?

   Hint: the secant line must be a linear function of the form $y = m_{\text{sec}}x + b$, and $P(1,3)$ must be on its graph. This information is sufficient to determine b in terms of m_{sec}, and you just need to calculate the slope m_{sec} of the line through P and Q.

[1] Stefan Sint, `sint@maths.tcd.ie`, see also `http://www.maths.tcd.ie/~sint/1S11.html`