MA1S11 Calculus, Tutorial Sheet 2[1]

19-21 October 2011

Useful facts:

- **Inverse**: A function $f(x)$ has an *inverse* if and only if it is strictly monotonically increasing or strictly monotonically decreasing. This means that it is either going up, $f(x_1) > f(x_2)$ whenever $x_1 > x_2$, or it is going down, $f(x_1) < f(x_2)$ whenever $x_1 > x_2$. If you draw a horizontal line anywhere on the graph of an invertible function, it will only cut it once.
- **Working out the inverse**. Let $y = f(x)$ and solve for x to get $x = f^{-1}(y)$ and exchange x and y. For instance, if $f(x) = x^3$ let $y = x^3$ hence $x = \sqrt[3]{y} = f^{-1}(y)$ so $f^{-1}(x) = \sqrt[3]{x}$.
- **Symmetry of curves**. A curve in the xy plane is symmetric
  1. about the x-axis if for any point (x, y) on the curve also $(x, -y)$ is on the curve.
  2. about the y-axis if for any point (x, y) on the curve also $(-x, y)$ is on the curve.
  3. about the origin if for any point (x, y) on the curve also $(-x, -y)$ is on the curve.

Questions
The numbers in brackets give the numbers of marks available for the question.

1. (2) We define the function $f(x) = \sqrt{x^3 - 1}$. What is its natural domain? Is this function invertible? If so, what is its inverse function? Also give its domain and range?
2. (3) Determine the symmetry properties (symmetry about x-axis, about y-axis, about the origin or none at all) of the following curves in the xy plane (you do not need to draw the graphs!):
$$y^2 = 3x^2, \qquad y^3 = x^7 + \sin(x), \qquad x^2 + \frac{y^2}{4} = 1, \qquad y^4 = x^{-3} + x.$$
3. (3) Consider the functions
$$f(x) = 3x + 1, \qquad g(x) = 2\sqrt{x + 1}.$$
What are their natural domains and their ranges? Determine their inverse functions $f^{-1}(x)$ and $g^{-1}(x)$. and graph them together with f and g.

[1] Stefan Sint, `sint@maths.tcd.ie`, see also `http://www.maths.tcd.ie/~sint/1S11.html`