## MA1S11 Tutorial Sheet $1^1$

## 12-14 October 2011

## Useful facts:

- Domain and range: The *domain* of a function f(x) is the set of x-values it is defined for, the *range* is the set of values f(x) when x is varied over its domain. If no domain is given explicitly, the *natural domain* is the set of all x-values for which f(x) is defined and real.
- Composition of functions: Given 2 functions f and g we define the *composition* of f with g by

$$(f \circ g)(x) = f(g(x)).$$

Obviously, x must be in the domain of g and g(x) in the domain of f for this definition to make sense.

• sum, difference, product, quotient of functions: Given functions f and g one defines new functions:

$$(f \pm g)(x) = f(x) \pm g(x),$$
  $(fg)(x) = f(x)g(x),$   $(f/g)(x) = f(x)/g(x),$ 

where x must be in the domain of both f and g and for the quotient f/g one needs to exclude all those x for which g(x) = 0.

• Even and odd functions: A function f(x) is even if f(x) = f(-x) for all x, it is odd if f(x) = -f(-x)

## Questions

The numbers in parenthesis give the numbers of marks available for the question.

- 1. (1) What is the natural domain and the range of  $f(x) = \sqrt{x^2 2x 3}$ , where you can note the factorization:  $x^2 2x 3 = (x + 1)(x 3)$ .
- 2. (1) What is the natural domain of  $f(x) = \sqrt{x+1}/(x^2-1)$ ?
- 3. (1) Suppose f and g are odd functions. Show that their sum f + g is an odd function and that their product fg is an even function.
- 4. (1) Determine whether each of the following is even or odd or neither and show why:  $f(x) = 1/x^3$ ,  $f(x) = \cos(x+1)$ ,  $f(x) = \sin x^3$ ,  $f(x) = x^4 + 5$

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- 5. (2) For any function f(x) whose domain is the real numbers show g(x) = f(x) + f(-x) is even and h(x) = f(x) f(-x) is odd. Show that any function whose domain is the real numbers can be written as the sum of an even and an odd function. Work out g and h for  $f(x) = x^2 x + 1$ .
- 6. (2) Consider the piecewise defined function

$$f(x) = \begin{cases} x & x < -2 \\ 0 & -2 \le x < 0 \\ x^2 & x \ge 0 \end{cases}$$

- (a) (1) Sketch the graph of f(x), f(-x) and -f(x).
- (b) (1) Sketch the graph of f(x) + 1, f(x + 1), f(2x) and 2f(x).