MA2321: Exercises 7, 2011

23 November 2011

1. Let v be a tangent vector at a and f and g be scalar fields at a on a manifold. Show that
$$v(fg) = f(a)vg + g(a)vf$$

2. Let v be a vector field and f and g be scalar fields on a manifold. Denote by vf the scalar field $(vf)(a) = v_a f$ Show that
$$v(fg) = fvg + gvf$$

3. Let f and g be scalar fields on a manifold. Show that
$$d(fg) = fdg + gdf$$

4. Let f be a scalar field and y^i be coordinates on a manifold. Show that df has components:
$$\frac{\partial f}{\partial y^i}$$

5. Let ω be a differential 1-form and v be a vector field on a manifold. Denote by $< \omega, v >$ the scalar field whose value at a is $< \omega_a, v_a >$.

   Calculate $< \omega, v >$ if ω has components ω_i and v has components v^i

6. Let x and y be coordinates and v a vector field on a 2-dimensional manifold, Find
$$[(dx)^2 + (dy)^2](v) = (< dx, v >)^2 + (< dy, v >)^2$$
   in terms of the components of v

   If $x = r\cos\theta$ and $y = r\sin\theta$, write $(dx)^2 + (dy)^2$ in terms of dr and $d\theta$.

7. Let x, y, z be coordinates and v a vector field on a 3-dimensional manifold, Find
$$[(dx)^2 + (dy)^2 + (dz)^2](v)$$
   in terms of the components of v

   If $x = r\sin\theta\cos\phi$ and $y = r\sin\theta\sin\phi$ and $z = r\cos\theta$, write $(dx)^2 + (dy)^2 + (dz)^2$ in terms of dr and $d\theta$ and $d\phi$.