MA2321: Exercises 6, 2011

16 November 2010

1. Find a necessary and sufficient condition that the functions

$$u = x + y \text{ and } v = xy$$

should be a coordinate system with domain an open neighbourhood of (a, b) in $\mathbf{R}^2$.

2. Let X be the subset of $\mathbf{R}^5$ given by the equations

$$F(x, y, z, t, u) = 0$$

$$G(x, y, z, t, u) = 0$$

where F and G are C^∞ functions on an open set V in $\mathbf{R}^5$. Write down a sufficient condition for x, y, z to be coordinates on X at a given point (a, b, c, d, e) of X. Calculate $\dfrac{\partial t}{\partial x}$ on X with respect to these coordinates.

3. Show that the equation

$$xt + yz = 1$$

defines a 3-dimensional manifold in $\mathbf{R}^4$.

4. Let A be a real symmetric $n \times n$ matrix, let $k \neq 0$, and let $\mathbf{R}^n$ have its usual scalar product. Show that the equation

$$(Ax|x) = k$$

defines an $(n-1)$-dimensional manifold in $\mathbf{R}^n$.

5. Show that the quadric

$$3x^2 + 4y^2 + 5z^2 + 6xy + 8xz - 2yz = 4$$

is a manifold in $\mathbf{R}^3$.

6. Let f be the map from the space of real $n \times n$ matrices to the space of real symmetric $n \times n$ matrices, given by: $f(A) = A^t A$.

   Recall that if A is an orthogonal matrix then the linear operator $f'(A)$ is surjective.

   Deduce that the set of real orthogonal matrices is a manifold, and find its dimension.