MA2321 Exercises 5; 2011

2 November 2011

1. Give an example of a collection of open subsets of $\mathbf{R^2}$ whose intersection is not open in $\mathbf{R^2}$.

2. Let S^1 be the set of points in $\mathbf{R^2}$ satisfying the equation $x^2+y^2=1$. Show that the set V of points in S^1 satisfying the equation $y>0$ is open in S^1. Show that V is homeomorphic to an open set in $\mathbf{R}$.

   Show that there is a C^∞ function f of one real variable such that $y=f(x)$ on V.

3. Let S^2 be the set of points in $\mathbf{R^3}$ satisfying the equation $x^2+y^2+z^2=1$. Show that the set V of points in S^2 satisfying the equation $z>0$ is open in S^2. Show that V is homeomorphic to an open set in $\mathbf{R^2}$.

   Show that there is a C^∞ function f of two independent real variables such that $z=f(x,y)$ on V.

4. Let X be the set of points in $\mathbf{R^2}$ satisfying $y=f(x)$ where f is a continuous real valued function on an open set V in $\mathbf{R}$. Show that X is homeomorphic to V.

5. Let X be the set of points in $\mathbf{R^3}$ satisfying $z=f(x,y)$ where f is a continuous real valued function on an open set V in $\mathbf{R^2}$. Show that X is homeomorphic to V