MA2322 Exercises 4; 2012

8 February 2012

1. Let $x_1, ..., x_r \in M$. Show that $x_1, ..., x_r$ are linearly independent if and only if

$$x_1 \wedge ..... \wedge x_r \neq 0$$

2. Let $x_1, ..., x_r$ be a basis for a vector subspace N of M. Let $y \in M$. Show that $y \in N$ if and only if

$$y \wedge x_1 \wedge ..... \wedge x_r = 0$$

3. Let $x_1, ..., x_r$ be linearly independent. Show that $x_1, ..., x_r$ generate the same subspace as $y_1, ..., y_r$ if and only if: $x_1 \wedge ..... \wedge x_r$ is a scalar multiple of $y_1 \wedge ..... \wedge y_r$

4. Let vectors $x_1, ..., x_r$ be linear combinations of the vectors $y_1, ..., y_n$ with

$$x_j = \alpha^i_j y_i$$

summed i from 1 to n.

Show that

$$x_1 \wedge ... \wedge x_r$$

is equal to

$$\sum \det A^{i_1,\ldots,i_r}_{1,\ldots,r} y_{i_1} \wedge ... y_{i_r}$$

summed over all increasing sequences $1 \leq i_1 < ... < i_r \leq n$ where $A^{i_1,\ldots,i_r}_{1,\ldots,r}$ denotes the $r \times r$ matrix formed by the $(i_1, ..., i_r)-$ rows of $A = (\alpha^i_j)$.

5. Let T be a linear operator from M to N, both finite dimensional. Show that the push-forward T_* acting on $\wedge^r M$ can be represented by a matrix whose entries are $r \times r$ determinants.

6. Let T be a linear operator of rank r from M to N. Show that the T_* acting on $\wedge^s M$ is nonzero for all $s \leq r$ and is zero for all $s > r$.

7. Let T be a linear operator with matrix A with respect to some basis. Show that T has rank r if and only if A has a nonzero $r \times r$ subdeterminant and all its $(r+1) \times (r+1)$ subdeterminants are zero.