MA2321 Exercises 4; 2011

26 October 2011

1. Let $\frac{\partial}{\partial x_a}$ be the operator on real valued functions differentiable at $a \in \mathbf{R}^2$ defined by

$$\frac{\partial}{\partial x_a} f = \frac{\partial f}{\partial x}(a)$$

Similarly for $\frac{\partial}{\partial y_a}$ and $\frac{\partial}{\partial r_a}$ and $\frac{\partial}{\partial \theta_a}$, where r, θ are polar coordinates. The real vector space generated by (spanned by) $\frac{\partial}{\partial x_a}$ and $\frac{\partial}{\partial y_a}$ is called the *tangent space of* $\mathbf{R}^2$ *at* a . Show that $\frac{\partial}{\partial x_a}, \frac{\partial}{\partial y_a}$ is a basis for the tangent space at a. Show that $\frac{\partial}{\partial r_a}, \frac{\partial}{\partial \theta_a}$ is also a basis for the tangent space at a.

2. Let f be a real valued differentiable function of two independent real variables. The equation $f(x, y) = 0$ is said to define y *implicitly* as a differentiable function F of x if $f(x, F(x)) = 0$. Find

$$\frac{dy}{dx} = F'(x)$$

in terms of the partial derivatives of f.

(Differentiate $f(x, F(x)) = 0$ with respect to x using the chain rule.)

3. Let f be a real valued differentiable function of three independent real variables. The equation

$$f(x, y, z) = 0$$

is said to define z *implicitly* as a differentiable function F of x and y if

$$f(x, y, F(x, y)) = 0$$

Find the partial derivatives of F in terms of the partial derivatives of f.

4. Let f and g be a real valued differentiable functions of three independent real variables. The equations

$$f(x, y, z) = 0, \qquad g(x, y, z) = 0$$

are said to define y and z *implicitly* as a differentiable functions F of x and G of x respectively if

$$f(x, F(x), G(x)) = 0$$

and

$$g(x, F(x), G(x)) = 0$$

Find the derivative of F in terms of the partial derivatives of f and of g.