MA2321 Exercises 3; 2014

9 October 2014

1. Let $u = f(x^3, \sin x, \cos x)$ where f is a differentiable function of three independent real variables. Find $\dfrac{du}{dx}$ in terms of the partial derivatives of f.

2. Show that if f is a C^2 function of one variable then $u = f(x - ct)$ is a solution of the equation
$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0.$$

3. Let f and g be functions of two independent real variables and suppose that
$$f(x, y) = F(g(x, y))$$
where F is a function of one independent real variable. Assume that the functi ons f, g, F are all differentiable. Prove that the Jacobian:
$$\frac{\partial(f, g)}{\partial(x, y)}$$
is zero.

4. Let f be a real valued differentiable function of two independent real variables. The equation $f(x, y) = 0$ is said to define y *implicitly* as a differentiable function F of x if $f(x, F(x)) = 0$. Find
$$\frac{dy}{dx} = F'(x)$$
in terms of the partial derivatives of f.

   (Differentiate $f(x, F(x)) = 0$ with respect to x using the chain rule.)

5. Let f be a real valued differentiable function of three independent real variables. The equation
$$f(x, y, z) = 0$$
is said to define z *implicitly* as a differentiable function F of x and y if
$$f(x, y, F(x, y)) = 0$$
Find the partial derivatives of F in terms of the partial derivatives of f.

6. Let f and g be a real valued differentiable functions of three independent real variables. The equations

$$f(x, y, z) = 0, \qquad g(x, y, z) = 0$$

are said to define y and z *implicitly* as a differentiable functions F of x and G of x respectively if

$$f(x, F(x), G(x)) = 0$$

and

$$g(x, F(x), G(x)) = 0$$

Find the derivative of F in terms of the partial derivatives of f and of g.