MA2321 Exercises 3; 2011

12 October 2011

1. Let $f : \mathbf{R}^2 \to \mathbf{R}$ be the function

$$f(x, y) = \frac{2xy^2}{x^2 + y^4}$$

if $(x, y) \neq (0, 0)$ and

$$f(0, 0) = 0.$$

Calculate:

(a) $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at $(0, 0)$

(b)

$$\frac{d}{dt} f(ta, tb)$$

at $t = 0$ for $a, b \in \mathbf{R}$ from first principles without using the chain rule.

Does the chain rule apply?

2. Let $u = f(x^3, \sin x, \cos x)$ where f is a differentiable function of three independent real variables. Find $\dfrac{du}{dx}$ in terms of the partial derivatives of f.

3. Show that if f is a C^2 function of one variable then $u = f(x - ct)$ is a solution of the equation

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} = 0.$$

4. Let f and g be functions of two independent real variables and suppose that

$$f(x, y) = F(g(x, y))$$

where F is a function of one independent real variable. Assume that the functions f, g, F are all differentiable. Prove that the Jacobian:

$$\frac{\partial(f, g)}{\partial(x, y)}$$

is zero.