MA2321 Exercises 2; 2014

3 October 2014

1. A function F of n real variables is called *homogeneous of degree r* if it satisfies
$$F(tx_1, tx_2, \ldots, tx_n) = t^r F(x_1, x_2, \ldots, x_n).$$
By differentiation with respect to t, show that such a function F is an eigenfunction of the operator:
$$x^1 \frac{\partial}{\partial x^1} + x^2 \frac{\partial}{\partial x^2} + \ldots + x^n \frac{\partial}{\partial x^n}$$
and find the eigenvalue.
$$x^1, \ldots\ldots, x^n$$
denote the usual coordinate functions on $\mathbf{R}^n$

2. Let $f : \mathbf{R}^2 \to \mathbf{R}$ be the function
$$f(x, y) = 2xy \frac{x^2 - y^2}{x^2 + y^2}$$
if $(x, y) \neq (0, 0)$ and
$$f(0, 0) = 0.$$
Show that
$$\frac{\partial^2 f}{\partial x \partial y} \neq \frac{\partial^2 f}{\partial y \partial x}$$
at $(0, 0)$.

3. Let $f : \mathbf{R}^2 \to \mathbf{R}$ be the function
$$f(x, y) = \frac{2xy^2}{x^2 + y^4}$$
if $(x, y) \neq (0, 0)$ and
$$f(0, 0) = 0.$$
Calculate $\frac{d}{dt} f(ta, tb)$ at $t = 0$ for $a, b \in \mathbf{R}$.

   Hence show that f cannot be differentiable at $(0, 0)$.

   If f was differentiable at $(0, 0)$, what would the chain rule give applied to $\frac{d}{dt} f(ta, tb)$ at $t = 0$?

   Deduce that the chain rule does not hold at $t = 0$.