

MA2321 Exercises 2; 2011

5 October 2011

1. Prove the following properties of the operator norm:

(a) $\|S + T\| \leq \|S\| + \|T\|$

(b) $\|Tx\| \leq \|T\|\|x\|$

(c) $\|TS\| \leq \|T\|\|S\|$

2. A function F of n real variables is called *homogeneous of degree r* if it satisfies

$$F(tx_1, tx_2, \dots, tx_n) = t^r F(x_1, x_2, \dots, x_n).$$

By differentiation with respect to t , show that such a function F is an eigenfunction of the operator:

$$x^1 \frac{\partial}{\partial x^1} + x^2 \frac{\partial}{\partial x^2} + \dots + x^n \frac{\partial}{\partial x^n}$$

and find the eigenvalue.

$$x^1, \dots, x^n$$

denote the usual coordinate functions on \mathbf{R}^n

3. Let $f : \mathbf{R}^2 \rightarrow \mathbf{R}$ be the function

$$f(x, y) = 2xy \frac{x^2 - y^2}{x^2 + y^2}$$

if $(x, y) \neq (0, 0)$ and

$$f(0, 0) = 0.$$

Show that

$$\frac{\partial^2 f}{\partial x \partial y} \neq \frac{\partial^2 f}{\partial y \partial x}$$

at $(0, 0)$.