MA2321 Exercises 2; 2009

7 October 2009

1. A function F of n real variables is called *homogeneous of degree r* if it satisfies

$$F(tx_1, tx_2, \ldots, tx_n) = t^r F(x_1, x_2, \ldots, x_n).$$

By differentiation with respect to t, show that such a function F is an eigenfunction of the operator:

$$x^1 \frac{\partial}{\partial x^1} + x^2 \frac{\partial}{\partial x^2} + \ldots + x^n \frac{\partial}{\partial x^n}$$

and find the eigenvalue.

$$x^1, \ldots.., x^n$$

denote the usual coordinate functions on $\mathbf{R}^n$

2. Let $f : \mathbf{R}^2 \to \mathbf{R}$ be the function

$$f(x, y) = 2xy \frac{x^2 - y^2}{x^2 + y^2}$$

if $(x, y) \neq (0, 0)$ and

$$f(0, 0) = 0.$$

Show that

$$\frac{\partial^2 f}{\partial x \partial y} \neq \frac{\partial^2 f}{\partial y \partial x}$$

at $(0, 0)$.

3. Let $f : \mathbf{R}^2 \to \mathbf{R}$ be the function

$$f(x, y) = \frac{2xy^2}{x^2 + y^4}$$

if $(x, y) \neq (0, 0)$ and

$$f(0, 0) = 0.$$

Calculate $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at $(0, 0)$. Calculate $\frac{d}{dt} f(ta, tb)$ at $t = 0$ for $a, b \in \mathbf{R}$. Deduce that the chain rule does not hold at $(0, 0)$.