

MA2321 Exercises 1; 2011

27 September 2011

1. Let $f : M \rightarrow N$ be a constant function. Show that $f'(a)$ is zero, for each $a \in M$.
2. Let $f : M \rightarrow N$ be a linear function. Show that $f'(a)$ is equal to f for each $a \in M$. Show that $f''(a)$ is zero for each $a \in M$.
3. Let $f : \mathbf{R}^{n \times n} \rightarrow \mathbf{R}^{n \times n}$ be defined by $f(A) = A^2$. Prove that f is differentiable. Find the derivative of f .
4. Let V be the space of real non-singular $n \times n$ matrices. Let

$$f : V \rightarrow \mathbf{R}^{n \times n}$$

be given by $f(A) = A^{-1}$. Find $f'(A)$.

5. Let $f : \mathbf{R}^{n \times n} \rightarrow \mathbf{R}^{n \times n}$ be defined by $f(A) = A^t A$. Prove that f is differentiable. Find the derivative of f . Prove that if A is an orthogonal matrix then the image of the linear operator $f'(A)$ is the space of real symmetric $n \times n$ matrices.

Note

Go to <http://www.maths.tcd.ie/~simms/MA2321.html> and

1. Click on 'PDF file for Exercises 1' to get the exercises above.
2. Click on 'PDF file for Course 221 (Real and Complex Analysis)' and go to Chapter 4, pages 71-88 for background notes on differentiation.