

MA2321 Exercises 1; 2009

30 September 2009

1. Let $f : M \rightarrow N$ be a constant function. Show that $f'(a)$ is zero, for each $a \in M$.
2. Let $f : M \rightarrow N$ be a linear function. Show that $f'(a)$ is equal to f for each $a \in M$. Show that $f''(a)$ is zero for each $a \in M$.
3. Let $f : \mathbf{R}^{n \times n} \rightarrow \mathbf{R}^{n \times n}$ be defined by $f(A) = A^2$. Prove that f is differentiable. Find the derivative of f .
4. Let V be the space of real non-singular $n \times n$ matrices. Let

$$f : V \rightarrow \mathbf{R}^{n \times n}$$

be given by $f(A) = A^{-1}$. Find $f'(A)$.

5. Let $f : \mathbf{R}^{n \times n} \rightarrow \mathbf{R}^{n \times n}$ be defined by $f(A) = A^t A$. Prove that f is differentiable. Find the derivative of f . Prove that if A is an orthogonal matrix then the image of the linear operator $f'(A)$ is the space of real symmetric $n \times n$ matrices.