

Course 141: MECHANICS

Problem Set 7

Date Issued: January 23, 2008

Date due: January 30, 2008

Bonus Set: Each problem counts 5 points

1. Water in a rotating container of radius 50 mm is 30 mm lower in the centre than at the edge. Find the angular velocity of the container.
2. The water in a circular lake of radius 1 km in latitude 60° is at rest relative to the earth. Find the depth to which the centre is depressed relative to the shore by the effect of centrifugal force. For comparison, find the height by which the centre is raised by the curvature of the earth surface.

3. A particle of mass m is subject of two forces, a central force $\vec{F}_1$ and a frictional force $\vec{F}_2$, with

$$\vec{F}_1 = f(r)\hat{r}; \quad \vec{F}_2 = -c\vec{v}, \quad c > 0$$

If the particle initially has angular momentum J_0 about $\vec{r} = 0$, find the angular momentum for all subsequent times.

4. A particle of mass m is attached to the end of a light string of length l . The other end of the string is passed through a small hole, and is slowly pulled through it. The particle is originally spinning round the hole with angular velocity ω . Find the angular velocity when the length of the string has been reduced to $l/2$. Find also the tension of the string when its length is r and verify that the increase in kinetic energy is equal to the work done by the force pulling the string through the hole (neglect gravity).
5. Calculate the work done in taking a particle around the circle $x^2 + y^2 = a^2$, $z = 0$ if the force is **(a)** $\vec{F} = y\hat{i}$; **(b)** $\vec{F} = x\hat{i}$. (Use the parametrization $x = a \cos \theta$, $y = a \sin \theta$, $z = 0$)