# Course 141: MECHANICS

Problem Set 7

**Date Issued: December 5, 2007**
**Date due: January 14, 2008**

Bonus Set: Each problem counts 15 points

1. The tides:

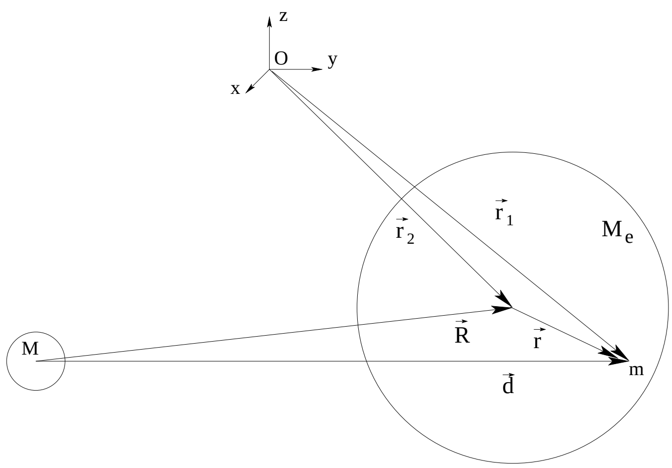
   (a) Assume earth is entirely covered by water. The ocean tides are due to the gravitational attraction of the moon and sun on the ocean surface. To see this, consider a mass $m$ on the ocean's surface and a distant mass $M$, as shown. It is not necessary to assume that $M$ is a point mass if $M$ is spherically symmetric. Let $M_e$ is the earth's mass. Write down the forces acting on $m$ and $M_e$. Show that (see figure)

   $$\ddot{\vec{r}} = -\frac{GM_e}{r^2}\hat{r} - GM\left(\frac{\hat{d}}{d^2} - \frac{\hat{R}}{r^2}\right)$$

   The first term is simply the earth's gravitational force per unit mass; the second is the tide generating force per unit mass, i.e., a mass $m$ of water on the ocean's surface experiences a tidal force

   $$\vec{F} = -mM\left(\frac{\hat{d}}{d^2} - \frac{\hat{R}}{R^2}\right)$$

   Perform the calculations in the inertial frame $O$ not associated with earth.




   (b) Assuming $M$ is above the equator, make a sketch of the magnitude and direction of the tidal forces on the ocean surface, North, South, East and in between.

2. A particle is dropped from a 100 $m$ tower on the equator. Where does the particle land? Do calculations in inertial frame. (Answer: 2.2 $cm$ Eastwards from tower's base)

3. Assume that the sun's density as a function of distance $r$ from its centre is described by

$$\rho(r) = \rho_c[1 - (r/R)^\alpha],$$

where $R$ is sun's radius, $\rho_c$ sun's central density, and $\alpha$ is to be determinated.

   (a) Find the sun's mass as a function of $r$, i.e., calculate

$$m(r) = 4\pi \int_0^r \rho(r')r'^2 dr'$$

   (b) Express $\rho_c$ in term of the sun's total mass $M$, raduiu $R$ and $\alpha$. It is known that $M = 1.99 \cdot 10^{30}\ kg$, $R = 6.96 \cdot 10^8\ m$. Calculate the average density of the sun, $\rho_{av}$, in $kg/m^3$. Express $\rho_c$ in terms of $\rho_{av}$ and $\alpha$.

   The sun is transforming hydrogen into helium in its core, and so the density there must be high. This requires $\alpha < 1$. Suppose $\alpha = 1$. What is the density ar $r = R/2$ in terms of $\rho_{av}$? Repeat this calculation with $\alpha = 0.1$ and $\alpha = 0.01$. You will see that $\alpha$ is a rapidly changing function of $\rho(R/2)$. Set $\alpha = 0.1$, then the density at $r = R/2$ is a little more than twice the sun's average density, while at $r = 0$ it is about 30 times $\rho_{av}$. For $\alpha = 0.1$ you should find $\rho_c = 4.3 \cdot 10^4\ kg/m^3$. Tne observed value is $1.6 \cdot 10^5\ kg/m^3$, so the choice $\alpha \sim 0.1$ is not bad! Make a sketch of $\rho(r)$ versus $r$.

   (c) Find a general formula for the pressure at the sun's centre using

$$\frac{dP}{dr} = -\frac{Gm(r)\rho(r)}{r^2}$$

   Express your answer in terms of $G, M, R$ and $\alpha$. Check that your answer is dimensionally correcr, i.e., force/area. Calculate the central pressure using the value of $\alpha$ in (b). (Hint: Obviously, the pressure on the sun's surface, $R(R) = 0$.)

   (d) Assuming the perfect gas law holds, i.e., $PV = NKT$ at the sun's centre (here $N$ is a number of particels in volume $V$), and the sun is mainly hydrogen, estimate the temperature of the sun's core. Assume all hydrogen at the sun's core is ionized. Compare your answer with the observed value $T \sim 1.5 \cdot 10^7 K^\circ$.

   Data: $k = 1.38 \cdot 10^-23\ N\ m/K$ (Boltzmann's constant), proton mass $\sim 1.67 \cdot 10^{-27}\ kg$.