Course 141: MECHANICS

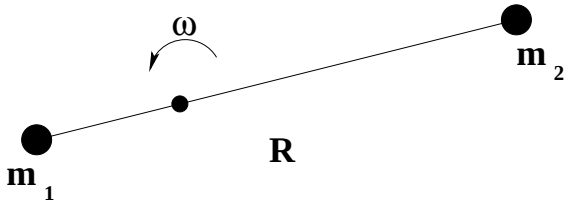
Problem Set 5

Date Issued: November 21, 2007
Date due: November 28, 2007

Each problem counts 10 points

1. A steel ball with mass 1 kg falls from a height of 3 m onto a pile of sand. The ball penetrated the sand a distance of 0.1 m before stopping. What constant force (in Newtons) has the sand exerted on the body?

2. A binary start is a common object composed of two stars of mass m_1 and m_2. Here we will assume they are separated by a constant distance R, and that they are rotating about a fixed point on a line joining them. Find their period of rotation T in terms of G, m_1, m_2 and R.



3. A particle of mass m is free to slide on a thin rod. The rod rotates in a plane about one end at constant angular velocity ω. Show that the motion is given by $r = Ae^{-\gamma t} + Be^{\gamma t}$, where γ is a constant which you must find, and A and B are two other constants. Neglect gravity.
   Show that for a particular choice of initial conditions, i.e., $r(t = 0)$ and $v(t = 0)$, it is possible to obtain a solution such that r decreases continually in time, but that for any other choice r will increase.

4. A block of mass m slides on a frictionless table. It is constrained to move inside a ring of radius r which is fixed to the table. At $t = 0$ the block is moving along the inside of the ring (i.e., in the tangential direction) with velocity v_0. The coefficient of frictions between the block and the ring is μ. Find the velocity of the block at later times.

   Hints: suppose a mass exerts a force normal to a surface of magnitude F. If the coefficient of friction between m and the surface is μ, the friction force opposing m's motion on the surface is μF. You will get a differential equation of the form $\frac{dv}{dt} = f(v)$. Put all the v's on pne side and dt on the other. Integrate, using the initial data given.