

Course 141: MECHANICS

Problem Set 18

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1. A uniform ball of mass M and radius R is pivoted so that it can turn freely about one of its diameters which is fixed in a vertical position. A beetle of mass m can crawl on the surface of the ball. Initially the ball is rotating with angular speed ω the beetle at the North pole. The beetle then walks to the “equator” of the ball and sits down. What is the angular velocity of the ball now?
2. Find the change of the kinetic energy of the system in Problem 1 caused by the beetle’s journey.
3. A cotton reel is at rest on a rough horizontal table when the free end of the thread is pulled horizontally with a constant force F . Given that the reel undergoes the planar motion, how does it move? Analyse the cases of the strong pull and the gentle pull.
4. In a circus trick, a performer of mass m causes a large ball of mass M and radius R to accelerate to the right by running to the left on the upper surface of the ball. The man does not fall off the ball because he maintains this motion in such a way that the angle α to the vertical direction remains constant. Find the conditions necessary for such a motion to take place.
5. A uniform ladder of length $2a$ is supported by a smooth horizontal floor and leans against a smooth vertical wall. The ladder is released from rest in a position making an angle 60° with the downward vertical. Find the energy conservation equation for the ladder.
6. A uniform rectangular plate has mass M and sides $2a$ and $2b$. Find the principal moments of inertia and the products of inertia in the coordinate system with the origin (a) at its centre of mass; (b) at a corner point.