Course 141: MECHANICS

Problem Set 17

Date Issued: April 16, 2008
Date due: April 23, 2008

1. (5 points) Find the moment of inertia about an axis through its centre of a uniform hollow sphere of mass M and outer and inner radii r_1 and r_2. Hint: Consider it as a sphere of density ρ and radius r_1, with a sphere of density ρ and radius r_2 removed.

2. (5 points) A thin uniform rod of mass M is supported by two vertical strings attached to its ends. Find the tension in the remaining string immediately after one of the strings is severed.

3. (10 points) A pendulum consists of two masses m_1 and m_2 connected by a very light rigid rod. The pendulum is free to oscillate in the vertical plane about a horizontal axis located at a distance r_1 from m_1 and at a distance r_2 from m_2.

   (a) Calculate the moment of inertia of the system about the axis. Find the location of the center of mass.

   (b) Set up the equation of motion for the system and derive the potential energy function.

   (c) Take $r_2 > r_1$ and determine the frequency of oscillations for small angles of displacement from the vertical.

   (d) Find the minimum angular velocity which must be given to the system (starting at equilibrium) if it is to continue in rotation instead of oscillating.

4. (5 points) A physical pendulum is made of a uniform disk of mass M and radius R suspended from a rod of negligible mass. The distance from the pivot to the center of the disc is l. What value of l makes the period a minimum?

5. (10 points) A ball of radius R rolling with velocity v on a level surface collides inelastically with a step of hight $h < R$. Find the minimum velocity for which the ball will "trip" up over the step. Assume that no slipping occurs at the impact point. Hint: Use the conservation of the total angular momentum about the point of impulsive contact.

6. (5 points) A physical pendulum consists of a solid cylinder which is free to rotate about a transverse axis displaced by a distance d along the symmetry axis from the center of mass. Find the value of d for which the period is a minimum. Express the result in terms of the mass M and moment of inertia I about a transverse axis through the center of mass.