Course 141: MECHANICS

Problem Set 15

Date Issued: March 5, 2008
Date due: April 2, 2008

1. (10 points) Suppose a hole is drilled through the earth, which does not necessarily pass through the earth's centre. Someone places you in a chair and drops you in. How long will it take you to the opposite end of the hole? Assume that the density of the earth is uniform and neglect the earth's rotation.

2. (5 points) Where is the centre of mass of the sun-Jupiter system? The mass ratio is $M_S/M_J = 1047$. Through what angle does the sun's position as seen from the earth oscillate because of the gravitational attraction of Jupiter?

3. (10 points) A particle of mass m moves under a central force described by the potential energy function $V(r) = \dfrac{a}{r^2} + br^2$ where a, c are positive constants.

   (a) Sketch this function, and decribe the possible types of motion.

   (b) Determine the position of the equilibrium r_0 and the frequency of small oscillations about it, that is, calculate the Teylor series of $V(r)$ about the equilibrium point r_0, including the term of order $(r - r_0)^2$.

4. (5 points) A proton is elastically scattered through an angle of 56° by a nucleus, which recoils at an angle of 60°. Find the atomic mass of the nucleus, and the fraction of the kinetic energy transferred to it.

5. (5 points) A mass m moving horizontally with velocity v_0 strikes a pendulum of mass m.

   (a) If two masses stick together, find the maximum height reached by the pendulum.

   (b) If the masses scatter elestically along the line of the initial motion, find the resulting maximum height.

6. (10 points) A steel ball-bearing is dropped from a height h onto a flat steel plate. The coefficient of restitution, i.e., the ratio of the relative velocity after collision to the relative velocity before collision, is ϵ. Find the total distance travelled by the ball-bearing untill it comes to rest and how long it bounces. (Hint: first analyse the case of one bounce to show that the new hight is $h_1 = \epsilon^2 h$ and the time up is $t_1 = \epsilon\sqrt{2h/g}$. Then apply this result successively.)