Course 141: MECHANICS

Problem Set 12

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1. A particle of mass m and electric charge q moves in a static Coulomb-like magnetic field $\vec{B} = g\frac{\hat{r}}{r^2}$ where g is a constant. The magnetic force on a particle is

$$\vec{F} = q\dot{\vec{r}} \times \vec{B}$$

   Find the conservation laws for this motion. Discuss qualitatively the character of motion.

2. A particle of mass m moves under an isotropic harmonic oscillator force with potential energy $V(r) = kr^2/2$. Initially it moves in a circle of radius a. Find the orbital velocity v. It is then given a blow of impulse mv in a direction making an angle α with its original velocity. Use the conservation laws to determine its minimum and maximum distances from the origin during the subsequent motion. Explain your results physically for the two limiting cases $\alpha = 0$ and $\alpha = \pi$.

3. The minimum distance of a comet from the sun is observed to be half the radius of the earth's orbit (assumed circular), and its velocity at that point is twice the orbital velocity of the earth. By using the conservation laws, find its velocity when it crosses the earth's orbit, and the angle at which the orbit cross. Will the comet subsequently escape from the solar system? What kind of orbit does it follow?

4. The sun has an orbital velocity of about 250 $km\ s^{-1}$ around the centre of the galaxy, whose distance is 30000 light years. Estimate the total mass of the galaxy in solar masses.

5. A particle of mass m moves under an attractive central force $\vec{F} = Cr^4\hat{r}$ with angular momentum J. For what energy will the motion be circular, and what is the radius of the circle?

6. For what values of n are circular orbits stable with the potential energy

$$U(r) = -A/r^n,$$

   where $A > 0$?