Course 141: MECHANICS

Problem Set 10

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1. The general solution of the equation of motion of a simple harmonic oscillator can be written in 3 different forms:

$$x = a\cos\omega t + b\sin\omega t; \quad x = A\cos(\omega t - \theta); \quad x = \mathbf{Re}\ (Ce^{i\omega t})$$

   A harmonic oscillator of angular frequency 2 s^{-1} is initially at $x = -3\ m$ with $\dot{x} = 8\ m\ s^{-1}$. Write the solution in each of the three forms. Find the first time at which $x = 0$ and $\dot{x} = 0$. Sketch the solution.

2. When a mass is suspended from a spring, the equilibrium length is increased by 50 mm. Given that the mass is then given a blow which starts it moving vertically at 200 $mm\ s^{-1}$, find the period and amplitude of the resulting oscillations, assuming neglidgible damping.

3. Find, which of the following forces are conservative, and for those that are find the corresponding potential energy function. Here a, b are constants, $\vec{a}$ is a constant vector:

   (a) $F_x = ax + by^2, \quad F_y = az + 2bxy, \quad F_z = ay + bz^2$

   (b) $F_x = ay, \quad F_y = az, \quad F_z = ax$

   (c) $F_r = 2ar\sin\theta\sin\phi, \quad F_\theta = ar\cos\theta\sin\phi, F_\phi = ar\cos\phi$

   (d) $\vec{F} = \vec{a} \times \vec{r}$

   (e) $\vec{F} = r\vec{a}$

   (f) $\vec{F} = \vec{a}(\vec{a} \cdot \vec{r})$

4. Given that the force is as in Problem 3(a), evaluate the work done in taking a particle from the origin to the point (1,1,0)

   (a) by moving first along the x-axis and the parallel to the y-axis;

   (b) by going in a straight line.

   Verufy that the result in each case is equal to minus the change in the potential energy function.

5. Evaluate the force corresponding to the potential energy function $V(\vec{r}) = cz/r^3$, where c is a constant. Write your answer in vector notations, and also in spherical polar coordinats, and verify that it satisfies $\vec{\nabla} \times \vec{F} = 0$.