# The Hall Effect

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## Contents

# 1 Abstract

In this experiment the characteristics of an *Electromagnet* were investigated, and then this electromagnet was used to show that the *Hall Voltage* $V_H$ is directly proportional to the product of the magnetic field $B$ times the current $I$, and to then measure the *Hall Coefficient* $R_H$ for a Germanium Crystal (Ge), which was found to be $-0.0313 \pm 0.006\text{m}^3\text{C}^{-1}$.

The *Carrier Type* and concentration $N$, the electrical conductivity of the sample $\sigma$ and the *Carrier Mobility* were also measured. These were found to be electrons with $N = 1.99 \pm 0.04 \times 10^{20}\text{m}^{-3}$ and $\sigma = 22.2 \pm 0.5\text{S m}^{-1}$ respectively, and finally $\mu$ was found to be $1.44\pm0.04\text{m}^2\,\text{V}^{-1}\,\text{s}^{-1}$.

These values of $R_H$ and $N$ are in agreement with the accepted order of magnitudes for an n type Germanium crystal semiconductor at room temperature.

Using a Halbach magic cylinder the average value for the carrier concentration for a different germanium crystal was found to be $5.5\pm0.1\times10^{20}\text{m}^{-3}$. The average value of the Hall coefficient was measured to be $0.0113\pm0.0002\text{m}^3\text{C}^{-1}$ and sign of the Hall coefficient was positive corresponding to a carrier type of holes and the sample is a p-type germanium crystal.

The average values calculated for the conductivity $\sigma$ and carrier mobility $\mu$ were $740\pm70\text{S m}^{-1}$ and $0.84\pm0.10\text{m}^2\,\text{V}^{-1}\,\text{s}^{-1}$ respectively. These values are of the same order of magnitude as the expected values for a p-type germanium crystal at room temperature.

Finally, the properties and uses of a lock-in-amplifier were investigated. It was seen the the output voltage was a maximum for a phase shift of $\theta = 20 \pm 5^o$ corresponding to a minimum at $\theta = 115 \pm 5^o$ as expected. Again a value for the Hall coefficient was calculated to be $0.010\pm0.001\text{m}^3\text{C}^{-1}$ in agreement with the value found using the magic cylinder.

# 2 Introduction & Theory

## 2.1 The Hall Effect

If a meal or semiconductor is placed in an external magnetic field $B$ and a current density $J$ is passed through the sample, a transverse electric field $E_H$ is set up given by

$$E_H = R_H B \times J \qquad (1)$$

where $R_H$ is called the Hall Coefficient, and this process is know as the Hall Effect. Charges $q$ moving is the semiconductor with velocity $v$ will experience the Lorentz force

$$F = qv \times B$$

as well as a force due to the transverse electric field

$$F = qE_H$$

The electric field will increase until the forces on these charges balance. Then, noting that

$$J = Nqv$$

where N is the number of charges, from equation 1 we can get we get the expression

$$R_H = \frac{1}{Nq} \qquad (2)$$

Finally, for a crystal with dimensions $l$, $w$ and $t$ respectively, and on seeing that

$$V_H = E_H w$$

and

$$I = wtJ$$

again from equation 1 we get the expression

$$V_H = \frac{R_H}{t} BI \quad (3)$$

## 2.2 Carrier Type and Mobility

In a metal or semiconductor we may have either electrons or positive holes acting as the charge carriers. However, for the same current and magnetic field direction the direction of $E_H$ will be opposite for either holes or electrons, and thus the sign of $R_H$ depends on the charge carrier type. If $R_H$ is known, the Carrier Concentration may be $N$ may be found from equation 2. With this, the Carrier Mobility may be found by measuring the Electrical Conductivity $\sigma$ using

$$\sigma = Nq\mu \quad (4)$$

on noting that

$$\sigma = \frac{1}{\rho}$$

where the Resistivity $\rho$ is given by the resistance between two terminals through the relation

$$R_{12} = \frac{\rho l}{wt} \quad (5)$$

## 2.3 The Halbach Magic Cylinder

A *Magic Cylinder* is a device made of several permanent magnets placed in a circle such that their direction of magnetisation causes a resultant transverse magnetic filed through the centre. In the limit as the number of domains and length of the cylinder go to infinity the resultant field is uniform. For a cylinder of sufficient length and number of segments the magnetic field may be approximated as uniform however.

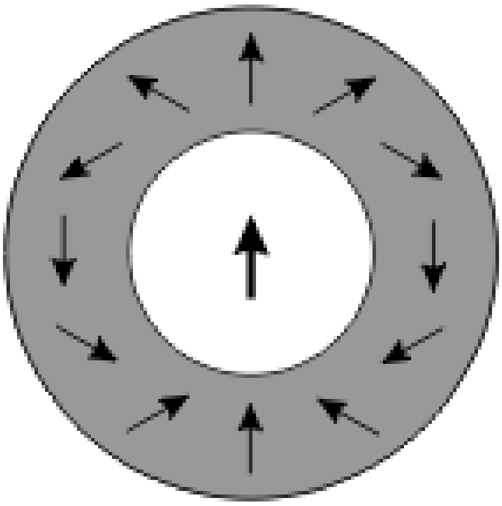

Figure 1: The Halbach Magic Cylinder Resultant Field

## 2.4 The Lock-In-Amplifier

A Lock-In-Amplifier may be used to increase the signal-to-noise ratio in measurements by several orders of magnitude by focusing on a narrow bandwidth around the frequency to be measured. As noise occurs on a wide bandwidth it can thus be blocked out.

# 3 Experimental Method

## 3.1 The Electromagnetic

The electromagnet circuit was set up as follows

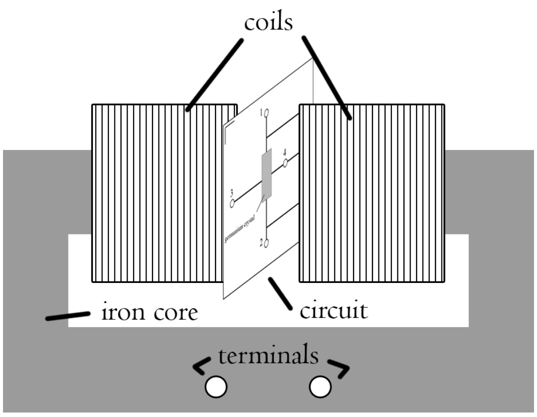


Figure 2: Electromagnet Circuit

where the germanium crystal circuit is given by

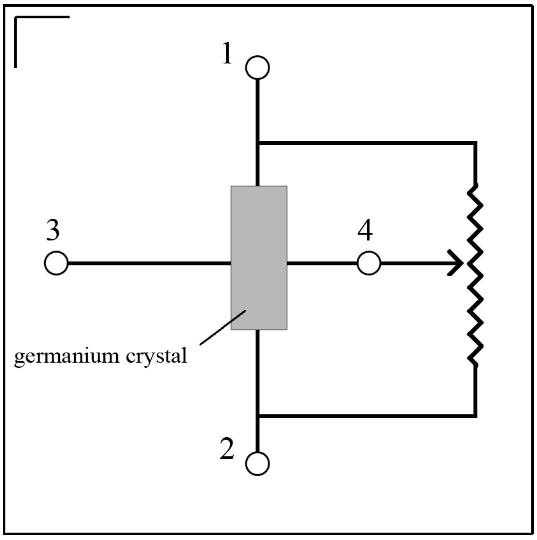


Figure 3: Ge Crystal Circuit

A current $I_C$ was passed through the electromagnet. The Hall probe Gaussmeter was inserted, and was rotated until a maximum reading was found in order to have the correct orientation.

A compass was then used to find the direction of the $B$ field.

The magnetic flux density $B$ was then measured for a range of values of current $I$ between -2.5A and +2.5A, letting it vary from 0A to +2.5A, +2.5A to 0A, 0A to -2.5A, and finally from -2.5A to 0A, taking note only to reverse $I_C$ the current was zero.

A graph of $B$ versus $I_C$ was then plotted, and was used to determine the *Remnant Field* $B_r$.

Finally, the $B$ field was measured as the current $I_C$ varied between -1A and +1A.

## 3.2 The Hall Coefficient

The circuit was set up to allow current to flow through the germanium crystal, putting a 1k$\Omega$ resistor in series with the sample and the 50V power supply.

The current was set to 10mA, and the magnetic field was set to 0, so that by adjusting the potentiometer the voltage difference $V_0$ due to misalignment of the terminals was minimised.

The voltage across terminals 3 and 4, $V_{34}$ was measured for a range of values of $B$ three times, with currents of 10mA, 20mA and 30mA flowing respectively. A graph of $V_{34}$ versus B was plotted each for each of the three cases.

The graph was then used to calculate the Hall Coefficient $R_H$ using equation (3).

From $R_H$, the Carrier Concentration $N$ was then calculated using equation (2).

The sign of $R_H$ was then determined using the direction of the $B$ field found above.

The Conductivity $\sigma$ was then found by measuring the voltage drop $V_{12}$ and by then using equation (5).

Finally, the Carrier Mobility $\mu$ was calculated using equation (4).

## 3.3 The Halbach Magic Cylinder

The Germanium crystal circuit was placed in the magic cylinder, without the potentiometer.

The resultant magnetic field **B** was set perpendicular to the sample by finding a maximum or minimum.

A current of 5 m A was passed through terminals 1 to 2, and the voltages $V_{12}$ and $V_{34}$ across terminals 1 to 2 and 3 to 4 respectively were measured for both directions of **B**.

This was repeated for a range of values of current $I$.

The Hall voltage $V_H$ and the misalignment voltage $V_0$ were found using

$$V_{34} = V_H + V_0$$

where

$$V_H = 1/2[V_{34}(B, I) - V_{34}(-B, I)]$$

and

$$V_0 = 1/2[V_{34}(B, I) + V_{34}(-B, I)]$$

The carrier concentration $N$, Hall coefficient $R_H$, carrier type, the sample conductivity $\sigma$ and the carrier mobility $\mu$ were calculated using equations (2), (3), (5) and (4) respectively and the sign of $R_H$ was determined.

Finally the misalignment of terminals 3 and 4 was found.

The circuit was then connected to an oscilloscope which was set to DC Coupled so that $V_0$ may be measured.

The cylinder was set continuously rotating and $V_34$ was measured for both directions of **B** off the oscilloscope.

The Hall coefficient $R_H$, the sample conductivity $\sigma$ and the carrier mobility $\mu$ were again measured and the misalignment of terminals 3 and 4 was calculated using equations (3), (5) and (4) respectively.

### 3.4 The Lock-In-Amplifier

The magic cylinder circuit was then connected to the lock-in-amplifier.

The rotation frequency of the magic cylinder was used as the reference frequency for the lock-in-amplifier.

The gain was set to minimum, and the offset was turned off. Then, for a current $I = 10\,\text{mA}$ the gain was slowly increased until a suitable value was found.

The output voltage $V_{out}$ was recorded for a range of phase shift angles $\theta$ and a graph of $V_{out}$ versus $\theta$ was plotted.

The value of $\theta$ for maximum output voltage was determined from the graph.

The Hall coefficient $R_H$ and the carrier concentration $N$ were found using equations (3) and (2).

The lock-in-amplifier was adjusted to give zero direct current output, and then the phase was shifted by $90^o$ to give a maximum.

By this method the Hall voltage $V_H$ was measured for a range of values of current $I$ with maximum direct current.

## 4 Results & Analysis

### 4.1 The Electromagnet

From the data obtained, a graph of the magnetic field $B$ versus the current $I_C$ was plotted as follows

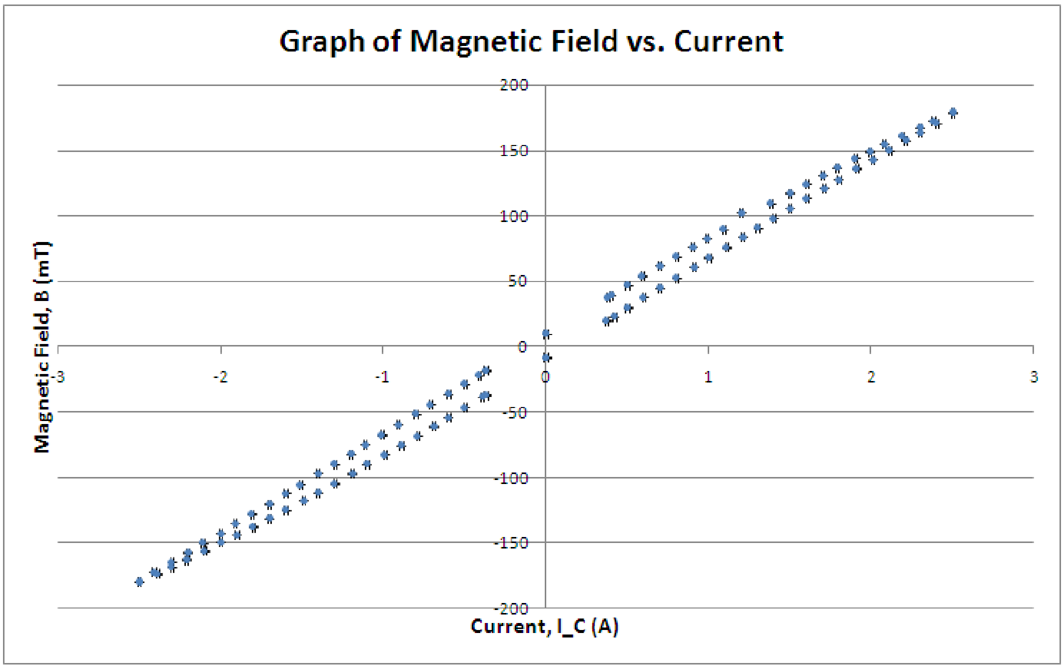


Figure 4: Magnetic Field versus Current

When the current $I_C$ was varied from -1A to +1A the following data was recorded

| Current $I_C$, ±0.1A | Magnetic Flux Density $B$, ±0.001mT |
|---|---|
| -1 | 68.4 |
| 0 | 2.37 |
| 1 | -69.9 |
| 0 | -5.17 |

## 4.2 The Hall Coefficient

The misalignment voltage $V_0$ was reduced to $0.1 \pm 0.1$mV for a magnetic field of $-0.60 \pm 0.005$mT. The dimensions of the germanium crystal $l$, $w$ and $t$ were 10mm, 5mm and 1mm respectively with an error of ±0.02mm on each. The following data was then obtained for the voltage $V_{34}$ as the magnetic field $B$ was varied, for currents of 10mA, 20mA and 30mA respectively.

| I=10mA | | I=20mA | | I=30mA | |
|---|---|---|---|---|---|
| Magnetic Field $B$, ±0.001mT | Voltage $V_{34}$, ±0.1mV | Magnetic Field $B$, ±0.001mT | Voltage $V_{34}$, ±0.1mV | Magnetic Field $B$, ±0.001mT | Voltage $V_{34}$, ±0.1mV |
| -80.1 | 22.4 | -68.9 | -46.2 | -69.2 | -72.3 |
| -77.2 | 21.8 | -66.5 | -44.8 | -66.7 | -69.9 |
| -63.9 | -20.8 | -63.7 | -43.1 | -61.3 | -68 |
| -61.8 | -20.1 | -60.7 | -41.3 | -59 | -65.1 |
| -59.3 | -19.3 | -59.8 | -40.6 | -57.2 | -62.9 |
| -56.8 | -18.6 | -56.9 | -38.3 | -55.2 | -59.4 |
| -54.8 | -17.9 | -55 | -37.4 | -52 | -56.4 |
| -51.6 | -16.9 | -51.5 | -35.5 | -50.2 | -54.6 |
| -49.7 | -16.4 | -50.3 | -34.7 | -47.5 | -52.1 |
| -47.5 | -15.6 | -47.7 | -33 | -45.3 | -50 |
| -44.9 | -14.8 | -45.01 | -31.4 | -43 | -47.9 |
| -42.5 | -14.1 | -41 | -28.8 | -40.3 | -45.2 |
| -39.6 | -13.1 | -39 | -27.5 | -37.3 | -39.6 |
| -37.4 | -12.4 | -37.2 | -26.3 | -34.4 | -37.3 |
| -34.6 | -11.6 | -34.6 | -24.7 | -29 | -34.3 |
| -31.7 | -10.6 | -31.8 | -22.8 | -26.5 | -31.9 |
| -28.4 | -9.5 | -28.6 | -20.8 | -23.9 | -29.3 |
| -26.3 | -8.8 | -26.1 | -19.2 | -21 | -26.6 |
| -23.5 | -7.9 | -23.3 | -17.4 | -17.96 | -23.6 |
| -20.2 | -6.8 | -20.1 | -15.3 | -15.04 | -20.8 |
| -17.62 | -6 | -17.86 | -13.9 | -13.42 | -19.2 |
| -15.21 | -5.2 | -14.84 | -11.9 | -3.19 | -9.2 |
| -13.6 | -4.7 | -13.49 | -11.1 | 6.73 | 0.5 |
| -3.24 | -1.4 | -3.19 | -4.3 | 9.19 | 2.6 |
| 7.01 | 1.9 | 7.02 | 2.1 | 11.42 | 4.7 |
| 8.69 | 2.4 | 9.04 | 3.4 | 13.68 | 7 |
| 11.14 | 3.2 | 11.54 | 5 | 17.42 | 10.4 |
| 14.9 | 4.3 | 14.52 | 6.9 | 20.43 | 13.2 |
| 17.35 | 5.1 | 17.46 | 8.7 | 24.49 | 17 |
| 20.34 | 6 | 20.87 | 10.9 | 26.35 | 18.7 |
| 23.35 | 7 | 23.33 | 12.4 | 28.83 | 21 |
| 26.7 | 8 | 26.13 | 14.1 | 31.8 | 23.8 |
| 29.46 | 8.8 | 28.83 | 16.1 | 34.7 | 26.4 |
| 32.1 | 9.7 | 31.8 | 17.7 | 37.7 | 29.1 |
| 34.6 | 10.4 | 34.7 | 19.4 | 40.9 | 31.9 |
| 38.1 | 11.5 | 37.7 | 21.2 | 42.9 | 33.7 |
| 40.9 | 12.4 | 41.5 | 23.4 | 46.7 | 37.1 |
| 44.5 | 13.4 | 44 | 24.9 | 48.8 | 38.9 |
| 46.9 | 14.1 | 46.9 | 26.7 | 52.3 | 41.8 |
| 49.9 | 15 | 49.6 | 28.2 | 56.1 | 45.1 |
| 52.4 | 15.7 | 52.5 | 29.9 | 58.6 | 47.2 |
| 55.2 | 16.5 | 55 | 31.3 | 60.6 | 48.7 |
| 58.4 | 17.4 | 57.6 | 32.7 | 63 | 50.7 |
| 60.5 | 18 | 61.5 | 34.9 | 65.1 | 52.5 |
| 63.4 | 18.7 | 63.3 | 35.8 | 68.6 | 55.3 |
| 65.8 | 19.4 | 66.3 | 37.5 | | |
| 69 | 20.3 | 7 68.8 | 38.8 | | |

This allowed us to plot the following graphs

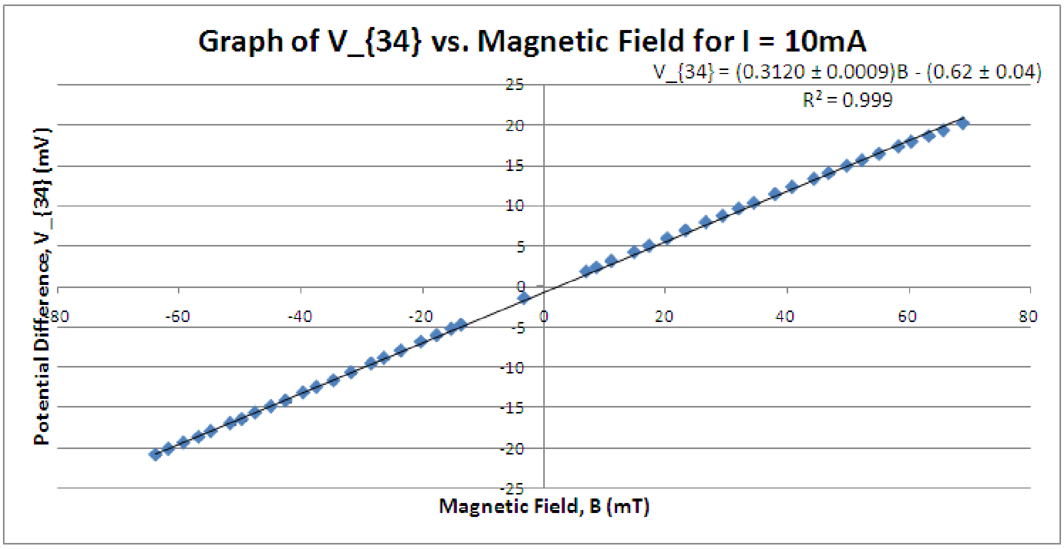


Figure 5: Potential Difference versus Magnetic Field

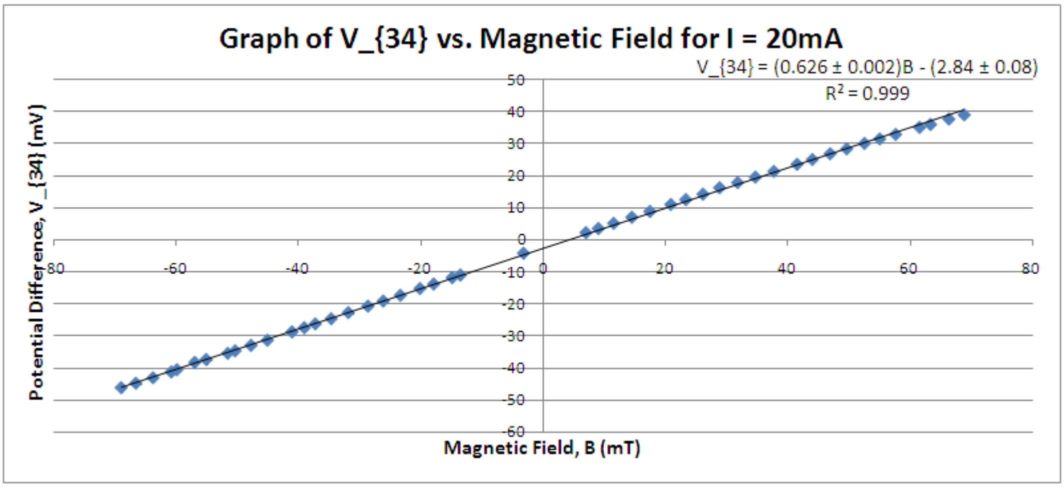


Figure 6: Potential Difference versus Magnetic Field

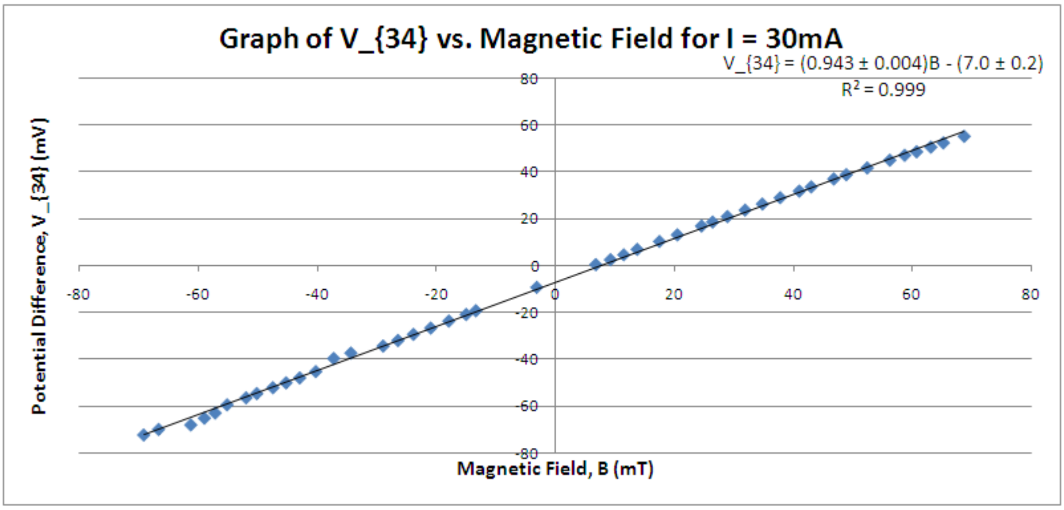


Figure 7: Potential Difference versus Magnetic Field

To measure the conductivity the following data was recorded for the potential difference $V_{12}$ as the current $I$ was varied

| Potential Difference $V_{12}$, $\pm 0.01$V | Current $I$, $\pm 0.1$mA |
|---|---|
| 5 | 0.443 |
| 10 | 0.886 |
| 15 | 1.323 |
| 20 | 1.767 |
| 25 | 2.22 |
| 30 | 2.67 |
| 35 | 3.13 |
| 40 | 3.6 |

and the following graph was then plotted

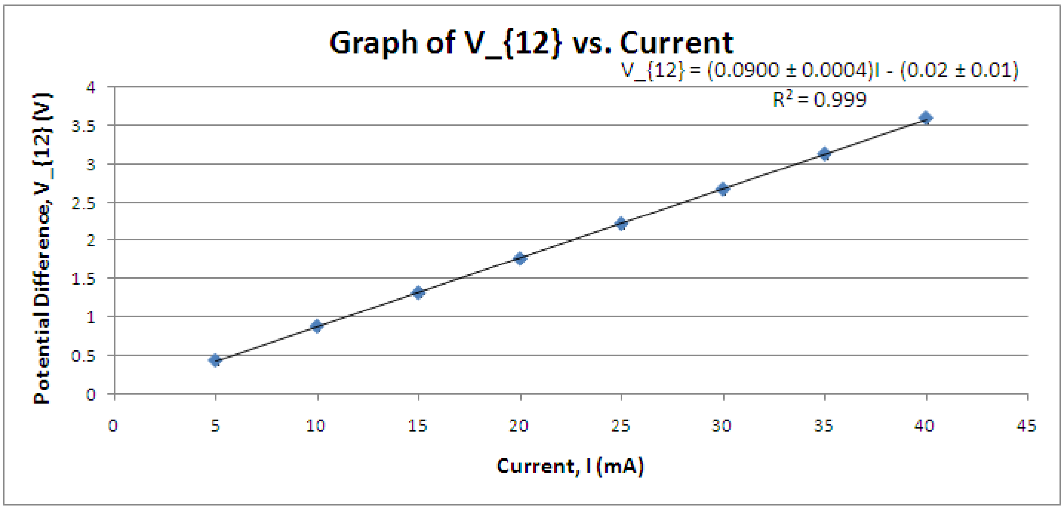


Figure 8: Potential Difference versus Current

## 4.3 The Halbach Magic Cylinder

The following data was obtained using the magic cylinder

| Current $I, \pm 0.01$mA | Voltage $V_{34}(B,I), \pm 0.1$mV | $V_{34}(-B,I)$, $\pm 0.1$mV | $V_{12}$, $\pm 0.1$V | Hall Voltage $V_H$, $\pm 0.1$V |
|---|---|---|---|---|
| 5 | 21.8 | 2.1 | 1.3 | 9.9 |
| 10 | 45.5 | 6.1 | 2.0 | 19.7 |
| 15 | 70.5 | 12.0 | 2.7 | 29.3 |
| 20 | 95.9 | 19.8 | 3.3 | 38.1 |
| 25 | 122.4 | 28.6 | 3.9 | 46.9 |
| Current $I, \pm 0.01$mA | Misalignment Voltage $V_0$, $\pm 0.1$mV | $V_0/V_{12}$ | Hall Coefficient $R_H$ ($\mathrm{m^3\,C^{-1}}$) | Resistance R ($\Omega$) |
| 5 | 12.0 | 9.1±0.7 | 0.0116±0.0007 | 2.39±0.02 |
| 10 | 25.8 | 12.8±0.6 | 0.0116±0.0007 | 2.58±0.01 |
| 15 | 41.3 | 15.4±0.5 | 0.0115±0.0007 | 2.750±0.007 |
| 20 | 57.9 | 17.4±0.5 | 0.0111±0.0007 | 2.890±0.005 |
| 25 | 75.5 | 19.2±0.5 | 0.0110±0.0007 | 3.020±0.004 |

The following values were then calculated using this data

| Current $I$, $\pm 0.01$mA | Carrier Concentration $N$, $\mathrm{m^{-3}} \times 10^{18}$ | Conductivity $\sigma$, ($\mathrm{Si\,m^{-1}}$) | Carrier Mobility $\mu$, ($\mathrm{m^2\,V^{-1}\,s^{-1}}$ |
|---|---|---|---|
| 5 | 540±30 | 840±20 | 0.97± 0.06 |
| 10 | 540±30 | 780±20 | 0.90± 0.06 |
| 15 | 540±30 | 730±20 | 0.83± 0.05 |
| 20 | 560±40 | 690±10 | 0.77± 0.05 |
| 25 | 570±40 | 660±10 | 0.73± 0.05 |

## 4.4 The Lock-In-Amplifier

The following graphs were plotted from the data obtained

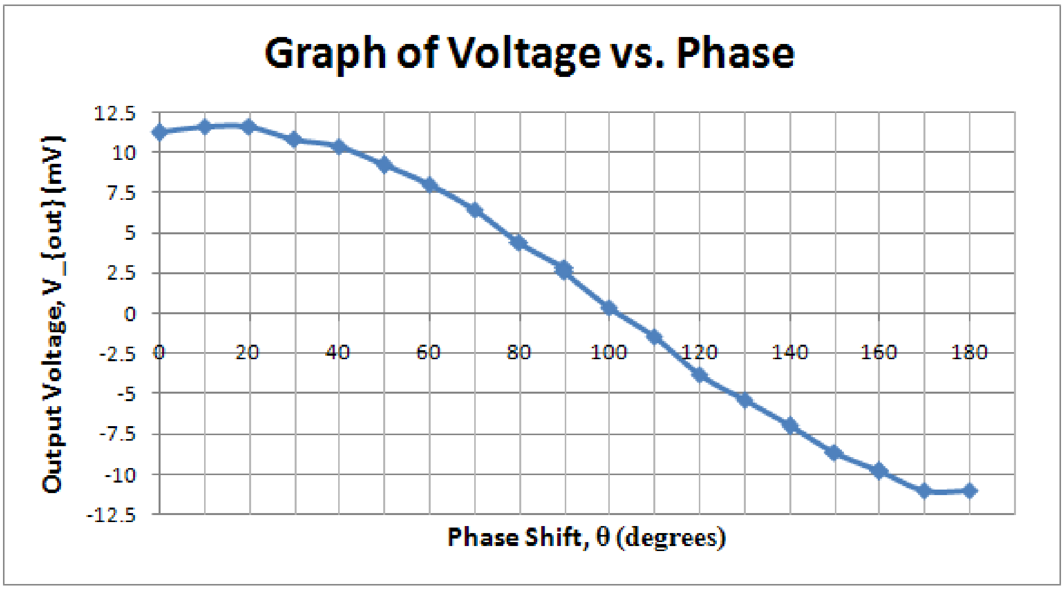


Figure 9: Graph of The Output Voltage versus The Phase Shift

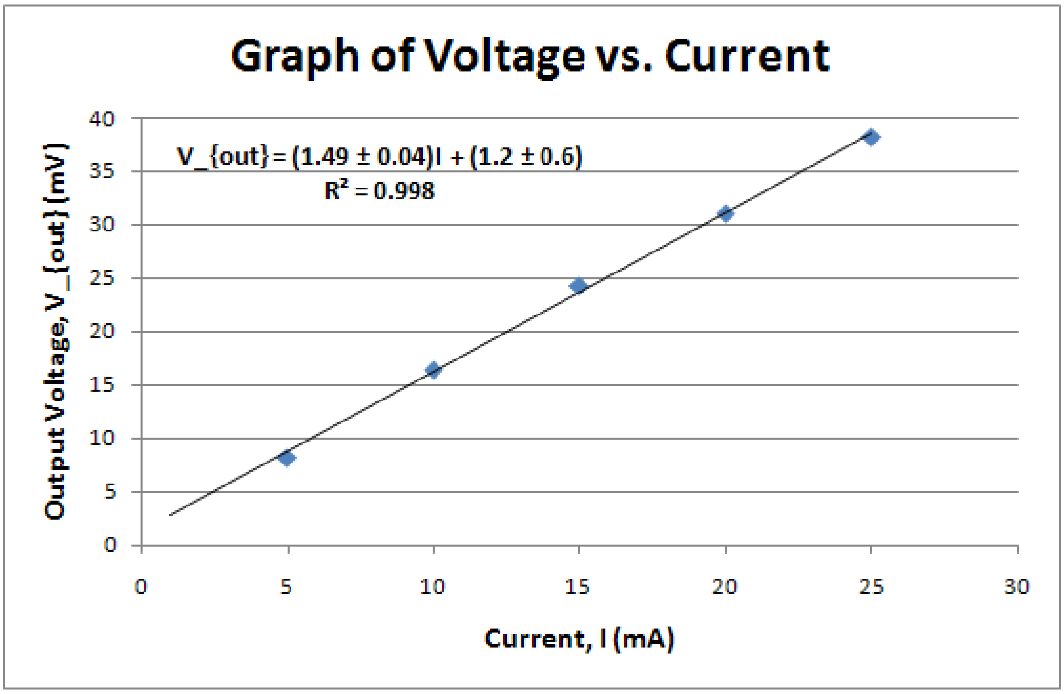


Figure 10: Graph of The Output Voltage versus The Current

# 5 Error Analysis

On taking the average value of $R_H$ from the three graphs the error in $R_H$ was calculated using the standard deviation. The errors in $\rho$, $\sigma$, $N$ and $\mu$ were calculated as follows

$$\Delta\rho = \rho \times \sqrt{\left(\frac{\Delta w}{w}\right)^2 + \left(\frac{\Delta t}{t}\right)^2 + \left(\frac{\Delta R_{12}}{R_{12}}\right)^2 + \left(\frac{\Delta l}{l}\right)^2}$$

$$\Delta\sigma = \sigma \times \sqrt{\left(\frac{\Delta\rho}{\rho}\right)^2}$$

$$\Delta N = N \times \sqrt{\left(\frac{\Delta R_H}{R_H}\right)^2}$$

$$\Delta\mu = \mu \times \sqrt{\left(\frac{\Delta\sigma}{\sigma}\right)^2 + \left(\frac{\Delta N}{N}\right)^2}$$

# 6 Conclusions

## 6.1 The Electromagnet

For the graph of the magnetic field $B$ versus the current $I_C$ a *Hysteresis* curve was found, and it was seen that there are two curves, one for when $I_C$ was varied from -2.5A to +2.5A, and one for when $I_C$ was varied from +2.5A back down to -2.5A. This is the Remnant Field $B_r$, and it is seen that there is no unique value for $B$ for a given $I_C$.

Furthermore, on varying the current between -1A and +1A, the curve for $B$ versus $I_C$ would be different, and it was concluded that the Remnant Field is dependent on the maximum value of $I_C$.

## 6.2 The Hall Coefficient

For the graphs of the potential difference $V_{34}$ versus the magnetic field $B$ we get straight line graphs through the origin and it was seen that $V_H \propto BI$, where $I$ was the current through the crystal.

Using the graphs, a value of $-0.0313 \pm 0.006\text{m}^3\text{C}^{-1}$ was found for the Hall Coefficient $R_H$, which was the same for each $I$. The direction of the magnetic was found to be in the negative x direction, which means that the sign of $R_H$ was negative, and thus that the charge carriers were electrons.

Using this value for $R_H$, a value of $1.99 \pm 0.04 \times 10^{20}\text{m}^{-3}$ was calculated for the Carrier Concentration $N$.

The Conductivity $\sigma$ of the sample was measured to be $22.2 \pm 0.5\text{S m}^{-1}$, which gave a value of $1.44 \pm 0.04\,\text{m}^2\,\text{V}^{-1}\,\text{s}^{-1}$ for the Carrier Mobility $\mu$.

These values of $R_H$ and $N$ are in agreement with the accepted order of magnitudes of $10^{-2}$ and $10^{20}$ for Germanium at room temperature[1].

## 6.3 The Halbach Magic Cylinder

Using the magic cylinder the average value for the carrier concentration for a different germanium crystal was found to be $5.5 \pm 0.1 \times 10^{20}\text{m}^{-3}$. The average value of the Hall coefficient was measured to be $0.0113 \pm 0.0002\text{m}^3\text{C}^{-1}$ and sign of the Hall coefficient was positive. Hence the carrier type is holes and the sample is a p-type germanium crystal.

The average values calculated for the conductivity $\sigma$ and carrier mobility $\mu$ were $740 \pm 70\text{S m}^{-1}$ and $0.84 \pm 0.10\text{m}^2\,\text{V}^{-1}\,\text{s}^{-1}$ respectively. These values are of the same order of magnitude as the expected values for a p-type germanium crystal at room temperature.

[1] Lerner, Rita, G., Trigg, George, L., Concise Encyclopedia of Solid-State Physics, (Addison- Wesley Publishing Company, Inc., 1983).

Using the oscilloscope to take measurements the average value of the Hall coefficient $R_H$, the conductivity $\sigma$ and carrier mobility $\mu$ were found to be $0.012 \pm 0.000 \mathrm{m}^3 \mathrm{C}^{-1}$, $960 \pm 50 \mathrm{S\,m}^{-1}$ and $1.13 \pm 0.05 \mathrm{m}^2\,\mathrm{V}^{-1}\,\mathrm{s}^{-1}$. These values are all within the limits of experimental error of the independently measured values above. Again, the sign of $R_H$ was determined to be positive and so the carrier type is holes.

## 6.4 The Lock-In-Amplifier

From the graph plotted it was seen the the output voltage was a maximum for a phase shift of $\theta = 20 \pm 5^o$ corresponding to a minimum at $\theta = 115 \pm 5^o$ as expected.

The Hall coefficient was once again determined to be $0.010 \pm 0.001 \mathrm{m}^3 \mathrm{C}^{-1}$, and this value is of the same order of magnitude as those calculated above.