The Coincidence Method

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1 Abstract

In this experiment the absolute activity of a Cobalt-60 radioactive source was determined using the *Coincidence Counting Method*. It was found to be 10,300±130Bq.

The resolving time for a Geiger-Müller counter was measured to be $(4.4\pm0.3)\times10^{-6}$s.

Furthermore, the laminar density of Aluminium needed to block β-particles was found to be $1.20\pm0.10\text{kg m}^{-3}$ which is of the same order of magnitude as the accepted value of 1.08kg m^{-3}.

Finally, the efficiency of the Geiger-Müller counters at detecting γ-rays and β-particles were calculated to be 26±6% and 12±3% for γ-rays and 0.3±0.2% for β-particles respectively.

2 Introduction & Theory

2.1 Absolute Activity

The *Absolute Activity* of a radioactive source that emits two γ-rays can be measured using two detectors with efficiencies $\varepsilon_{\gamma1}$ and $\varepsilon_{\gamma2}$ respectively. The count rates will then be

$$N_{\gamma1} = 2A\varepsilon_{\gamma1} \text{ and } N_{\gamma2} = 2A\varepsilon_{\gamma2} \tag{1}$$

respectively, where A is the absolute activity.

Using a special coincidence counting circuit to measure the coincident count rate $N_{\gamma1,\gamma2}$ where

$$N_{\gamma1,\gamma2} = 2A\varepsilon_{\gamma1}\varepsilon_{\gamma2}$$

the expression for the absolute activity is

$$A = \frac{N_{\gamma1}N_{\gamma2}}{2N_{\gamma1,\gamma2}} \tag{2}$$

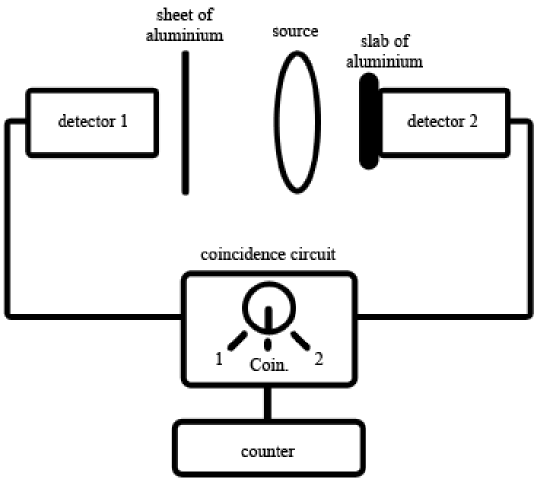

Figure 1: The Coincidence Counting Circuit

A Co^{60} source emits two γ-rays of energy 1.17MeV and 1.33MeV and a β-particle of energy 0.314MeV per disintegration and so the count rate will be larger. Using an Aluminium sheet the β-particles may be blocked from the detector. If an Aluminium sheet is not used, β-particles will also cause counts and the equation must be altered as now

$$N_{\beta\gamma1} = A(\varepsilon_{\beta1} + 2\varepsilon_{\gamma1})$$

so

$$N_{\beta\gamma1,\gamma2} = A(2\varepsilon_{\beta1}\varepsilon_{\gamma2} + 2\varepsilon_{\gamma1}\varepsilon_{\gamma2})$$

giving

$$A = \frac{(N_{\beta\gamma1} - 0.5N_{\gamma1})N_{\gamma2}}{N_{\beta\gamma1,\gamma2}} \tag{3}$$

where

$$\varepsilon_{\beta1} = \frac{N_{\beta\gamma1} - N_{\gamma1}}{A} \tag{4}$$

since $N_{\beta\gamma 1} - N_{\gamma 1}$ is the count rate for β-particles only.

2.2 The Resolving Time

If two particles from two separate decay events arrive one at each of the two detectors respectively within the *Coincidence Resolving Time* τ of one another then an *Accidental Count* will be recorded. The total number of accidental counts is given by

$$N_{\gamma 1,\gamma 2,\text{ acc}} = 2\tau N_{\gamma 1} N_{\gamma 2} \tag{5}$$

Using a single γ-ray source such as Cs the coincidence resolving time can be measured as any coincidence counts recorded will be accidental counts, and so

$$\tau = \frac{N^{(Cs)}_{\gamma 1,\gamma 2\text{ acc}}}{2N^{(Cs)}_{\gamma 1} N^{(Cs)}_{\gamma 2}} \tag{6}$$

The accidental coincidence count must be subtracted from all coincidence counts measured.

2.3 The Range of β-Particles

As β-particles pass through Aluminium they loose energy. For a certain thickness of an Aluminium sheet all β-particles will loose their energy and will not reach the detector.

2.4 Detector Efficiencies

Assuming that the Co^{60} acts as a point source the *Actual Efficiency* ε_{act} is the *Intrinsic Efficiency* ε_{int} times the number of particles entering the detector. This can be calculated as the ratio of the solid angle Ω subtended by the detector to the total solid angle of 4π, and so the intrinsic efficiency is given by

$$\begin{aligned} \varepsilon_{\text{int}} = \frac{4\pi}{\Omega}\varepsilon_{\text{act}} &= \frac{4\pi}{1}\left(\frac{\pi d^2}{r^2}\right)^{-1}\varepsilon_{\text{act}} \\ &= \frac{4d^2}{r^2}\varepsilon_{\text{act}} \end{aligned} \tag{7}$$

where $r = 8.5$mm is the radius of the detector and d is the distance from the Co^{60} source to the detector.

3 Experimental Method

3.1 Resolving Time

The single γ Cs source was used, and $N_{\gamma 1}$ and $N_{\gamma 2}$ were measured.

The circuit was set to "coin" and the accidental coincidence count $N_{\gamma 1,\gamma 2}$ was then measured.

The resolving time was calculated using equation (6).

3.2 The Range of β-Particles

The Co^{60} source was used.

The count rate was measured with no Aluminium sheet. This was then repeated using Aluminium sheets of a range of thickness.

A graph of the thickness versus the count rate was plotted and the β-particle range was found.

3.3 γ-γ Coincidence

Using the Co^{60} source, and the appropriate Aluminium sheet as determined above, $N_{\gamma 1}$ and $N_{\gamma 2}$ were measured.

The absolute activity was then calculated using equations (2) and (5).

3.4 $\beta\gamma$-γ Coincidence

The Aluminium sheet was removed, and the above experiment was repeated. The absolute activity was then found using equations (4) and (5).

3.5 Detector Efficiencies

The distance d from the Co^{60} source to the detector was measured.

The intrinsic efficiency of detector 1 for both γ-rays and β-particles $\varepsilon_{\gamma 1}$ and $\varepsilon_{\beta 1}$ and that of detector 2 for γ-rays $\varepsilon_{\gamma 2}$ were calculated using equations (1), (4) and (7).

4 Results & Analysis

4.1 Resolving Time

The following data was obtained for the Cs source

	Count Rate, R Bq
$N_{\gamma 1}^{Cs}$	26.9 ± 0.4
$N_{\gamma 2}^{Cs}$	9.6 ± 0.2
$N_{\gamma 1,\gamma 2}^{Cs}$	0.0023 ± 0.0002

4.2 The Range of β-Particles

The following graph was plotted using the data collected as the Aluminium sheet thickness x was varied

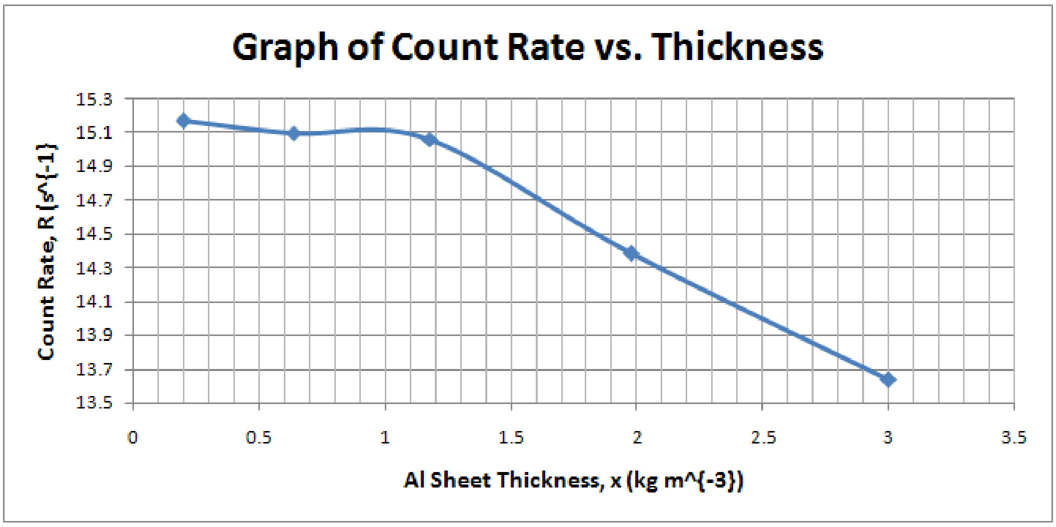

Figure 2: The Range of β-Particles versus Aluminium sheet Thickness

4.3 γ-γ Coincidence

The following data was obtained for the Co^{60} source using an Aluminium sheet of laminar density 5.2887 kg m^{-2}

	Count Rate, R Bq
$N_{\gamma 1}$	19.8 ± 0.1
$N_{\gamma 2}$	8.9 ± 0.2
$N_{\gamma 1,\gamma 2}$	0.010 ± 0.002

4.4 $\beta\gamma$-γ Coincidence

The following data was then obtained for the Co^{60} source on removing the Aluminium sheet

	Count Rate, R Bq
$N_{\beta\gamma 1}$	19.9 ± 0.2
$N_{\gamma 2}$	8.9 ± 0.2
$N_{\beta\gamma 1,\gamma 2}$	0.010 ± 0.002

4.5 Detector Efficiencies

The distance from the source to the detector d was measured to be 70 ± 1mm.

5 Error Analysis

The standard error in the time t was taken to be ± 1s for all counts. The standard error in the thickness of the Aluminium sheets x was taken to be $\pm 1\mu$m. The following equations were then used to calculate the respective errors

$$\Delta\text{Count} = \sqrt{\text{Count}}$$

$$\Delta\text{Rate} = \text{Rate} \times \sqrt{\left(\frac{\Delta\text{Count}}{\text{Count}}\right)^2 + \left(\frac{\Delta t}{t}\right)^2}$$

$$\Delta\tau = \tau \times \sqrt{\left(\frac{\Delta N^{(Cs)}_{\gamma1,\gamma2,\text{ acc}}}{N^{(Cs)}_{\gamma1,\gamma2,\text{ acc}}}\right)^2 + \left(\frac{\Delta N^{(Cs)}_{\gamma1}}{N^{(Cs)}_{\gamma1}}\right)^2 + \left(\frac{\Delta N^{(Cs)}_{\gamma2}}{N^{(Cs)}_{\gamma2}}\right)^2}$$

$$\Delta N_{\gamma1,\gamma2}\text{ acc} = A_{\gamma-\gamma}\text{ acc} \times \sqrt{\left(\frac{\Delta\tau}{\tau}\right)^2 + \left(\frac{\Delta N_{\gamma1}}{N_{\gamma1}}\right)^2 + \left(\frac{\Delta N_{\gamma2}}{N_{\gamma2}}\right)^2}$$

$$\Delta A_{\gamma-\gamma} = A_{\gamma-\gamma} \times \sqrt{\left(\frac{\Delta N_{\gamma1,\gamma2} - \Delta N_{\gamma1,\gamma2}\text{ acc}}{N_{\gamma1,\gamma2} - N_{\gamma1,\gamma2}\text{ acc}}\right)^2 + \left(\frac{\Delta N_{\gamma1}}{N_{\gamma1}}\right)^2 + \left(\frac{\Delta N_{\gamma2}}{N_{\gamma2}}\right)^2}$$

$$\Delta N_{\beta\gamma1,\gamma2}\text{ acc} = N_{\beta\gamma1,\gamma2}\text{ acc} \times \sqrt{\left(\frac{\Delta\tau}{\tau}\right)^2 + \left(\frac{\Delta N_{\beta\gamma1}}{N_{\beta\gamma1}}\right)^2 + \left(\frac{\Delta N_{\gamma2}}{N_{\gamma2}}\right)^2}$$

$$\Delta A_{\beta\gamma-\gamma} = A_{\beta\gamma-\gamma} \times \sqrt{\left(\frac{\Delta N_{\beta\gamma1,\gamma2} - \Delta N_{\beta\gamma1,\gamma2}\text{ acc}}{N_{\beta\gamma1,\gamma2} - N_{\beta\gamma1,\gamma2}\text{ acc}}\right)^2 + \left(\frac{\Delta N_{\gamma1}}{N_{\gamma1}}\right)^2 + \left(\frac{\Delta N_{\gamma2}}{N_{\gamma2}}\right)^2}$$

$$\Delta\Omega = \Omega \times \sqrt{2 \times \left(\frac{\Delta d}{d}\right)^2}$$

$$\Delta\varepsilon_j = \varepsilon_j \times \sqrt{\left(\frac{\Delta\text{Count}}{\text{Count}}\right)^2 + \left(\frac{\Delta A_i}{A_i}\right)^2}$$

$$\Delta\varepsilon_{j,\text{ int}} = \varepsilon_{j,\text{ int}} \times \sqrt{\left(\frac{\Delta\varepsilon_j}{\varepsilon_j}\right)^2 + \left(\frac{\Delta\Omega}{\Omega}\right)^2}$$

where τ is the resolving time, A_i are the respective absolute activities, Ω is the solid angle, d is the distance from the source to the detector and ε_j and $\varepsilon_{j,\text{int}}$ are the respective actual and intrinsic efficiencies for $j \in \{\gamma1, \gamma2, \beta\}$.

6 Conclusions

The resolving time of the detector was calculated to be $(4.4\pm0.3)\times10^{-6}$s.

It was found that β-particles were stopped by a sheet of Aluminium of laminar density of $1.20\pm0.10\text{kg}\,\text{m}^{-3}$ corresponding to a drop in count rate. This compares well with the accepted value of $1.08\text{kg}\,\text{m}^{-3}$ as it is of the correct order of magnitude[1].

The $\gamma-\gamma$ and $\beta\gamma-\gamma$ absolute activities were measured to be 10,400±2,300Bq and 10,200±1,800Bq respectively. These independently measured values are within the range of experimental error of each other, and thus verify each other. The average value for the absolute activity was calculated to be 10,300±130Bq.

Finally, the detector 1 intrinsic efficiencies for γ-rays and β-particles $\varepsilon_{\gamma1}$ and $\varepsilon_{\beta1}$ and the detector 2 intrinsic efficiency for γ-rays $\varepsilon_{\gamma2}$ were found to be 26±6%, 12±3% and 0.3±0.2% respectively.

[1]Kaye and Laby, textitTables of Physical and Chemical Constants, Longman Sc & Tech