1S11 (Timoney) Tutorial sheet 9

[November 27 – 30, 2012]

Name: Solutions

1. For

$$U = \begin{bmatrix} 1 & 1 & 3 & 5 \\ 0 & 1 & 4 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

find U^{-1}.

Solution: We can use the standard method of row reducing $[U|I_4]$ (to get $[I_4|U^{-1}]$).

$$\begin{bmatrix} 1 & 1 & 3 & 5 & :1 & 0 & 0 & 0 \\ 0 & 1 & 4 & -2 & :0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & :0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & :0 & 0 & 0 & 1 \end{bmatrix}$$

Add 2 times row 4 to row 2 and -5 times row 4 to row 1

$$\begin{bmatrix} 1 & 1 & 3 & 0 & :1 & 0 & 0 & -5 \\ 0 & 1 & 4 & 0 & :0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 & :0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & :0 & 0 & 0 & 1 \end{bmatrix}$$

Add -4 times row 3 to row 2 and -3 times row 3 to row 1

$$\begin{bmatrix} 1 & 1 & 0 & 0 & :1 & 0 & -3 & -5 \\ 0 & 1 & 0 & 0 & :0 & 1 & -4 & 2 \\ 0 & 0 & 1 & 0 & :0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & :0 & 0 & 0 & 1 \end{bmatrix}$$

Add -1 times row 2 to row 1

$$\begin{bmatrix} 1 & 0 & 0 & 0 & :1 & -1 & 1 & -7 \\ 0 & 1 & 0 & 0 & :0 & 1 & -4 & 2 \\ 0 & 0 & 1 & 0 & :0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & :0 & 0 & 0 & 1 \end{bmatrix}$$

Thus

$$U^{-1} = \begin{bmatrix} 1 & -1 & 1 & -7 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. For

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 0 & 3 & 0 & 1 \end{bmatrix}$$

find L^{-1}.

Solution: We can use the standard method again (of row reducing $[L|I_4]$).

$$\begin{bmatrix} 1 & 0 & 0 & 0 & :1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & :0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 & :0 & 0 & 1 & 0 \\ 0 & 3 & 0 & 1 & :0 & 0 & 0 & 1 \end{bmatrix}$$

Subtract 2 times row 1 from row 3

$$\begin{bmatrix} 1 & 0 & 0 & 0 & : & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & : & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & : & -2 & 0 & 1 & 0 \\ 0 & 3 & 0 & 1 & : & 0 & 0 & 0 & 1 \end{bmatrix}$$

Now subtract 3 times row 2 from row 4

$$\begin{bmatrix} 1 & 0 & 0 & 0 & : & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & : & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & : & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & : & 0 & -3 & 0 & 1 \end{bmatrix}$$

$$L^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ 0 & -3 & 0 & 1 \end{bmatrix}$$

3. For

$$A = \begin{bmatrix} 2 & 1 & 3 & 0 & 0 & 0 \\ 12 & -16 & 0 & -4 & 0 & 0 \\ 3 & -2 & 0 & 0 & 7 & 0 \\ -2 & 8 & 0 & 0 & 0 & -2 \\ 2 & 4 & 0 & 17 & 18 & -2 \\ 9 & 21 & 102 & -22 & 0 & -2 \end{bmatrix}$$

find $\mathrm{trace}(A)$ and find A^t.

Solution:

$$\mathrm{trace}(A) = 2 - 16 + 0 + 0 + 18 - 2 = 2$$

$$A^t = \begin{bmatrix} 2 & 12 & 3 & -2 & 2 & 9 \\ 1 & -16 & -2 & 8 & 4 & 21 \\ 3 & 0 & 0 & 0 & 0 & 102 \\ 0 & -4 & 0 & 0 & 17 & -22 \\ 0 & 0 & 7 & 0 & 18 & 0 \\ 0 & 0 & 0 & -2 & -2 & -2 \end{bmatrix}$$

4. Is this matrix nilpotent? (Explain your answer.)

$$A = \begin{bmatrix} 1 & 1/\sqrt{2} & 1 \\ 0 & 0 & -1/\sqrt{2} \\ 0 & -1/\sqrt{2} & -1 \end{bmatrix}$$

Solution: Recall that nilpotent means that some power is the zero matrix. We can try to find some powers of A:

$$\begin{aligned} A^2 &= \begin{bmatrix} 1 & 1/\sqrt{2} & 1 \\ 0 & 0 & -1/\sqrt{2} \\ 0 & -1/\sqrt{2} & -1 \end{bmatrix} \begin{bmatrix} 1 & 1/\sqrt{2} & 1 \\ 0 & 0 & -1/\sqrt{2} \\ 0 & -1/\sqrt{2} & -1 \end{bmatrix} \\ &= \begin{bmatrix} 1+0+0 & 1/\sqrt{2}+0-1/\sqrt{2} & 1-1/2-1 \\ 0+0+0 & 0+0+1/2 & 0+0+1/\sqrt{2} \\ 0+0+0 & 0+0+1/\sqrt{2} & 0+1/2+1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1/2 & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 3/2 \end{bmatrix} \\ A^3 &= AA^2 \\ &= \begin{bmatrix} 1 & * & * \\ 0 & * & * \\ 0 & * & * \end{bmatrix} \end{aligned}$$

(where the * means I have not worked out the numbers).

Since $A^3 \neq 0$ and A is 3×3, A is **not** nilpotent.

Another way: all powers of A will have the same (not zero) first column. So no power is zero and A is **not** nilpotent.

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