1S11 (Timoney) Tutorial sheet 8
[November 20 – 23, 2012]

Name: Solutions

1. For

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 0 & -2 \\ -4 & 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & 2 & 1 \\ 5 & -5 & 4 \\ 7 & -7 & 21 \end{bmatrix}, C = \begin{bmatrix} \frac{1}{2} & 3 \\ 2 & -1 \\ 1 & 0 \end{bmatrix}$$

compute

(a) the size of A (3×3) of B (3×3) and of C (3×2)

(b) the $(3, 2)$ entry of B

Solution: The $(3, 2)$ entry of B is -7.

(c) $5A + 2B$

Solution:

$$5A + 2B = \begin{bmatrix} 10 & -5 & 15 \\ 20 & 0 & -10 \\ -20 & 15 & 10 \end{bmatrix} + \begin{bmatrix} 0 & 4 & 2 \\ 10 & -10 & 8 \\ 14 & -14 & 42 \end{bmatrix} = \begin{bmatrix} 10 & -1 & 17 \\ 30 & -10 & -2 \\ -6 & 1 & 52 \end{bmatrix}$$

(d) AC

Solution: We need to take, one by one, the (dot) product of each row of A times each column of C

$$AC = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 0 & -2 \\ -4 & 3 & 2 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 3 \\ 2 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 7 \\ 0 & 12 \\ 6 & -15 \end{bmatrix}$$

2. For

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & -4 & -2 \\ -4 & 3 & -4 \end{bmatrix}$$

find the inverse matrix A^{-1} (using the method of row-reducing $[A|I_3]$ to reduced row-echelon form).

Solution: We use Gauss-Jordan elimination on

$$\begin{bmatrix} 2 & -1 & 3 & :1 & 0 & 0 \\ 4 & -4 & -2 & :0 & 1 & 0 \\ -4 & 3 & -4 & :0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{cccccc|} 1 & -\frac{1}{2} & \frac{3}{2} & :\frac{1}{2} & 0 & 0 \\ 4 & -4 & -2 & :0 & 1 & 0 \\ -4 & 3 & -4 & :0 & 0 & 1 \end{array}\right. \text{oldRow1} \times \frac{1}{2}$$

$$\left[\begin{array}{ccccccc} 1 & -\frac{1}{2} & \frac{3}{2} & : & \frac{1}{2} & 0 & 0 \\ 0 & -2 & -8 & : -2 & & 1 & 0 \\ 0 & 1 & 2 & : & 2 & 0 & 1 \end{array}\right| \begin{array}{l} \\ \text{oldRow2} - 4 \times \text{oldRow1} \\ \text{oldRow2} + 4 \times \text{oldRow1} \end{array}$$

$$\left[\begin{array}{ccccccc} 1 & -\frac{1}{2} & \frac{3}{2} & : & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 4 & : & 1 & -\frac{1}{2} & 0 \\ 0 & 1 & 2 & : & 2 & 0 & 1 \end{array}\right| \begin{array}{l} \\ \text{oldRow2} \times \left(-\frac{1}{2}\right) \\ \\ \end{array}$$

$$\left[\begin{array}{ccccccc} 1 & -\frac{1}{2} & \frac{3}{2} & : & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 4 & : & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & -2 & : & 1 & \frac{1}{2} & 1 \end{array}\right| \begin{array}{l} \\ \\ \text{OldRow3} - \text{OldRow2} \end{array}$$

$$\left[\begin{array}{ccccccc} 1 & -\frac{1}{2} & \frac{3}{2} & : & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 4 & : & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & : & -\frac{1}{2} & -\frac{1}{4} & -\frac{1}{2} \end{array}\right| \begin{array}{l} \\ \\ \text{OldRow3} \times \left(-\frac{1}{2}\right) \end{array}$$

$$\left[\begin{array}{ccccccc} 1 & -\frac{1}{2} & 0 & : & \frac{5}{4} & \frac{3}{8} & \frac{3}{4} \\ 0 & 1 & 0 & : & 3 & \frac{1}{2} & 2 \\ 0 & 0 & 1 & : & -\frac{1}{2} & -\frac{1}{4} & -\frac{1}{2} \end{array}\right| \begin{array}{l} \text{OldRow1} - \frac{3}{2} \times \text{OldRow3} \\ \text{OldRow2} - 4 \times \text{OldRow3} \\ \\ \end{array}$$

$$\left[\begin{array}{ccccccc} 1 & 0 & 0 & : & \frac{11}{4} & \frac{5}{8} & \frac{7}{4} \\ 0 & 1 & 0 & : & 3 & \frac{1}{2} & 2 \\ 0 & 0 & 1 & : & -\frac{1}{2} & -\frac{1}{4} & -\frac{1}{2} \end{array}\right| \begin{array}{l} \text{OldRow1} + \frac{1}{2} \times \text{OldRow2} \\ \\ \\ \end{array}$$

Thus the inverse matrix is

$$A^{-1} = \begin{bmatrix} \frac{11}{4} & \frac{5}{8} & \frac{7}{4} \\ 3 & \frac{1}{2} & 2 \\ -\frac{1}{2} & -\frac{1}{4} & -\frac{1}{2} \end{bmatrix}$$

Richard M. Timoney