

1S11 (Timoney) Tutorial sheet 4

[October 16 – 19, 2012]

Name: Solutions

1. Let $\mathbf{v} = -3\mathbf{i} + 4\mathbf{j} + 8\mathbf{k}$ and $\mathbf{w} = \mathbf{i} + 11\mathbf{j} + 14\mathbf{k}$. Find

(a) $\mathbf{v} \times \mathbf{w}$

Solution: Recall

$$\mathbf{v} \times \mathbf{w} = (v_2w_3 - v_3w_2)\mathbf{i} + (v_3w_1 - v_1w_3)\mathbf{j} + (v_1w_2 - v_2w_1)\mathbf{k}$$

(or write out table with 5 columns and last row $\mathbf{i}, \mathbf{j}, \mathbf{k}, \mathbf{i}, \mathbf{j}$). So

$$\begin{aligned}\mathbf{v} \times \mathbf{w} &= (56 - 88)\mathbf{i} + (8 + 42)\mathbf{j} + (-33 - 4)\mathbf{k} \\ &= -32\mathbf{i} + 50\mathbf{j} - 37\mathbf{k}\end{aligned}$$

(b) $\mathbf{w} \times \mathbf{v}$

Solution:

$$\mathbf{w} \times \mathbf{v} = -\mathbf{v} \times \mathbf{w} = 32\mathbf{i} - 50\mathbf{j} + 37\mathbf{k}$$

(c) $\mathbf{v} \cdot (\mathbf{w} \times \mathbf{v})$

Solution:

$$\mathbf{v} \cdot (\mathbf{w} \times \mathbf{v}) = 0$$

(general property of cross products — or compute $(-3)(32) + 4(-50) + 8(37) = 0$.)

(d) $(3\mathbf{v} + 4\mathbf{w}) \times (\mathbf{v} - \mathbf{w})$.

Solution:

$$\begin{aligned}(3\mathbf{v} + 4\mathbf{w}) \times (\mathbf{v} - \mathbf{w}) &= 3(\mathbf{v} \times \mathbf{v}) - 3(\mathbf{v} \times \mathbf{w}) \\ &\quad + 4(\mathbf{w} \times \mathbf{v}) - 4(\mathbf{w} \times \mathbf{w}) \\ &= \mathbf{0} - 3(\mathbf{v} \times \mathbf{w}) - 4(\mathbf{v} \times \mathbf{w}) + \mathbf{0} \\ &= -7(\mathbf{v} \times \mathbf{w}) \\ &= 224\mathbf{i} - 350\mathbf{j} + 259\mathbf{k}\end{aligned}$$

2. Use cross products to find a normal vector to the plane in \mathbb{R}^3 [*i.e.* in space] that contains the 3 points $(1, 4, 6)$, $(2, -1, 5)$ and $(7, -8, 2)$.

Solution: The vectors $2\mathbf{i} - \mathbf{j} + 5\mathbf{k} - (\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}) = \mathbf{i} - 5\mathbf{j} - \mathbf{k}$ and $\mathbf{i} - 8\mathbf{j} - 4\mathbf{k} - (\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}) = 6\mathbf{i} - 12\mathbf{j} - 4\mathbf{k}$ must lie in the plane. Thus their cross product is normal to the plane. That cross product is

$$(20 - 12)\mathbf{i} + (-6 + 4)\mathbf{j} + (-12 + 30)\mathbf{k} = 8\mathbf{i} - 2\mathbf{j} + 18\mathbf{k}$$

3. Find parametric equations for the line of intersection of the two planes

$$\begin{aligned}2x - 3y + 4z &= 1 \\x + 2y + 2z &= 3\end{aligned}$$

Solution: We need a vector parallel to the line (and also a point on the line) to write down the parametric equations.

We can think of the vector parallel as being along the line, hence perpendicular to the normal directions for each plane. So we want a vector perpendicular to both $2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ and $\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$. But the cross product of those two vectors will work just fine for that and so a vector parallel to the line is

$$(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}) \times (\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}) = (-6 - 8)\mathbf{i} + (4 - 4)\mathbf{j} + (4 + 3)\mathbf{k} = -14\mathbf{i} + 0\mathbf{j} + 7\mathbf{k}$$

So our line will have parametric equations of the form So our line will have parametric equations of the form

$$\begin{cases} x = x_0 - 14t \\ y = y_0 \\ z = z_0 + 7t \end{cases}$$

We need to find a point (x_0, y_0, z_0) on the line (any point on it). One way is to look for a point with $z = 0$. We then need x and y coordinates to satisfy

$$\begin{aligned}2x - 3y &= 1 \\x + 2y &= 3\end{aligned}$$

Add 2 times the first to 3 times the second to get $7x = 11$ or $x = 11/7$. Then $y = (3 - x)/2 = (21 - 11)/14 = 10/14 = 5/7$.

$$\begin{cases} x = \frac{11}{7} - 4t \\ y = \frac{5}{7} \\ z = 7t \end{cases}$$

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