1S11 (Timoney) Tutorial sheet 3

[October 9 – 12, 2012]

Name: Solutions

1. For $\mathbf{v} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{w} = 4\mathbf{i} - 5\mathbf{j} + \mathbf{k}$, find the projection $\mathrm{proj}_{\mathbf{w}}(\mathbf{v})$ along $\mathbf{w}$ of $\mathbf{v}$.

   Solution: Solution:

$$
\begin{aligned}
\mathrm{proj}_{\mathbf{w}}(\mathbf{v}) &= \frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{w}\|^2}\mathbf{w} \\
\mathbf{v}\cdot\mathbf{w} &= (3)(4) + (1)(-5) + 2(1) = 12 - 5 + 2 = 9 \\
\|\mathbf{w}\|^2 &= 4^2 + (-5)^2 + 1^2 = 16 + 25 + 1 = 42 \\
\mathrm{proj}_{\mathbf{w}}(\mathbf{v}) &= \frac{9}{42}\mathbf{w} \\
&= \frac{36}{42}\mathbf{i} - \frac{45}{42}\mathbf{j} + \frac{9}{42}\mathbf{k} \\
&= \frac{6}{7}\mathbf{i} - \frac{15}{14}\mathbf{j} + \frac{3}{14}\mathbf{k}
\end{aligned}
$$

2. Find the equation of the plane in space passing through the point $(1, 2, 3)$ perpendicular (normal) to the vector $6\mathbf{i} - 5\mathbf{j} + 4\mathbf{k}$.

   Solution: We know the equation has the form $6x - 5y + 4z = \text{const}$ (the coefficients of x, y and z are the compnents of the normal vector). Since $(1, 2, 3)$ is one point on the plane, we must have

$$6 - 10 + 12 = \text{const}$$

   and so the equation is $6x - 5y + 4z = 8$.

3. Find the equation of the plane in space passing through the points $(0, 2, 3)$, $(1, 0, 0)$ and $(2, 0, 0)$.

   Solution: We want an equation of the form $n_1x + n_2y + n_3z = c$ (where n_1, n_2 and n_3 are not all zero) and the information we have is that there are 3 values of (x, y, z) that satisfy the equations.

   Plugging each point in in turn says that we need n_1, n_2, n_3 and c to satisfy the three equations

$$
\begin{array}{rcrcrcl}
 & & 2n_2 & + & 3n_3 & = & c \\
n_1 & & & & & = & c \\
2n_1 & & & & & = & c
\end{array}
$$

   The last two equations look contradictory as we get both $n_1 = c$ and $n_1 = c/2$ from them — which must mean that $c = c/2$, so $c = 0$ and then $n_1 = 0$ also. So the equation

$n_1x + n_2y + n_3z = c$ must be of the form $n_2y + n_3z = 0$ and we need $2n_2 + 3n_3 = 0$. We still have two unknowns n_2 and n_3 and one equation for them. So we need

$$n_2 = -\frac{3}{2}n_3$$

and the equation of the plane has to be

$$-\frac{3}{2}n_3y + n_3z = 0$$

Any nonzero value of n_3 will be valid, or we can divide across by it and get

$$-\frac{3}{2}y + z = 0$$

as the equation of the plane.

(You might prefer to multiply both sides by -2 to get a tidier equation $3y - 2z = 0$ for the same plane.)

4. Find the equation of the plane through $(1, 2, 5)$ perpendicular to the line with parametric equations

$$\begin{aligned} x &= 3 + 2t \\ y &= 3 - 2t \\ z &= 1 + 7t. \end{aligned}$$

Solution: A vector parallel to the line is $2\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}$ and this is then normal to the plane. So the plane has equation $2x - 2y + 7z = d$ for some constant d.

As $(x, y, z) = (1, 2, 5)$ is on the plane we have to have $2 - 4 + 35 = d$ or $d = 33$ and the equation is $2x - 2y + 7z = 33$.

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