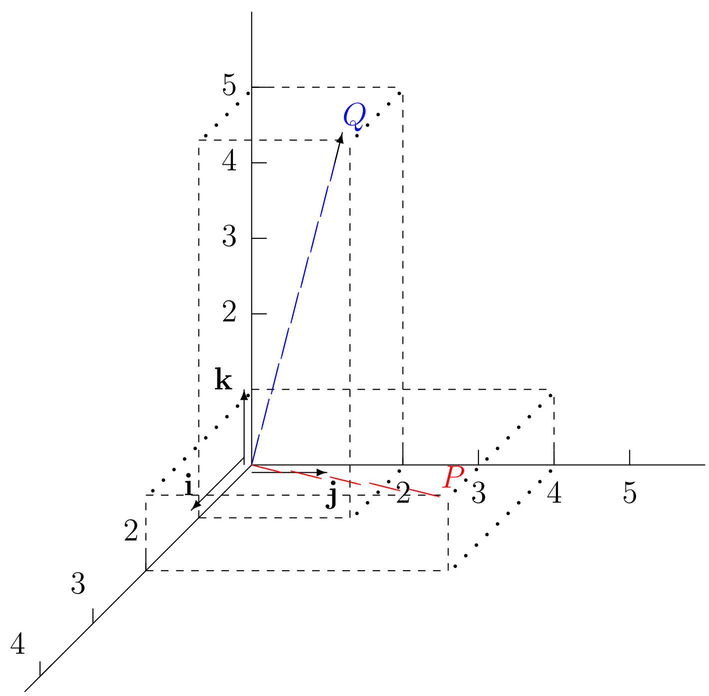
1S11 (Timoney) Tutorial sheet 2
[October 2 – 5, 2012]

Name: Solutions

1. (a) Show (on the graph) the point P with coordinates $(2, 4, 1)$ and the point Q with coordinates $(1, 2, 5)$.

   (b) Sketch the position vectors of the two points ($\mathbf{P}$ for P and $\mathbf{Q}$ for Q).



   (c) Calculate the distance from P to Q.

   Solution: By the distance formula

$$\begin{aligned}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} &= \sqrt{(1-2)^2 + (2-4)^2 + (5-1)^2} \\ &= \sqrt{1 + 4 + 16} = \sqrt{21}\end{aligned}$$

   is the distance from P to Q.

   (d) Calculate $\|\mathbf{Q} - \mathbf{P}\|$.

   Solution: As the vector $\mathbf{Q} - \mathbf{P}$ can be represented by an arrow from the point P to the point Q, the length of the vector must be exactly the distance from P to Q. We've just calculated that as $\sqrt{21}$.

2. For $\mathbf{v} = -3\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}$ and $\mathbf{w} = 6\mathbf{i} - 3\mathbf{j} + 7\mathbf{k}$, calculate

   (a) the cosine of the angle between $\mathbf{v}$ and $\mathbf{w}$
   Solution: We know $\mathbf{v}.\mathbf{w} = \|\mathbf{v}\|\|\mathbf{w}\|\cos\theta$.
   We need to calculate

$$\begin{aligned} \mathbf{v}.\mathbf{w} &= (-3)(6) + (7)(-3) + (2)(7) \\ &= -18 - 21 + 14 = -25 \\ \|\mathbf{v}\| &= \sqrt{(-3)^2 + 7^2 + 2^2} = \sqrt{62} \\ \|\mathbf{w}\| &= \sqrt{6^2 + (-3)^2 + 7^2} = \sqrt{94} \end{aligned}$$

   Then we have

$$-25 = \sqrt{62}\sqrt{94}\cos\theta$$

   from which we get

$$\cos\theta = \frac{-25}{\sqrt{62}\sqrt{94}}$$

   (b) the unit vector in the same direction as $\mathbf{w}$.
   Solution: The unit vector is $\mathbf{w}$ divided by its length $\|\mathbf{w}\|$, that is

$$\frac{1}{\|\mathbf{w}\|}\mathbf{w} = \frac{1}{\sqrt{94}}\mathbf{w} = \frac{6}{\sqrt{94}}\mathbf{i} - \frac{3}{\sqrt{94}}\mathbf{j} + \frac{7}{\sqrt{94}}\mathbf{k}$$

   (c) Is $\mathbf{v}$ perpendicular to $3\mathbf{i} - 7\mathbf{j} + 2\mathbf{k}$? (Why?)
   Solution:

$$\mathbf{v}.(3\mathbf{i} - 7\mathbf{j} + 2\mathbf{k}) = (-3)(3) + 7(-7) + 2(2) = -54 \neq 0$$

   and so not perpendicular.

3. Find the equation of the points (x, y, z) that are on the sphere of radius 2 and centre $(3, -4, 5)$. Find an answer without square roots. [Hint: the points on the sphere are those with distance from the center exactly equal to the radius.]

   Solution: If we write the description of the points on the sphere as a formula, we get

$$\begin{aligned} \text{distance}((x, y, z), (3, -4, 5)) &= 2 \\ \sqrt{(x-3)^2 + (y+4)^2 + (z-5)^2} &= 2 \\ (x-3)^2 + (y+4)^2 + (z-5)^2 &= 4 \end{aligned}$$

   We have squared both sides to eliminate the square root.

   Note: Normally we could introduce extra solutions to an equation by squaring both sides. The equation $t = 4$ has just the one solution, but $t^2 = 16$ has two solutions: $t = 4$ and $t = -4$. As the distance between points is never negative, we can find the distance if we know its square. So we do not introduce any extra solutions by squaring both sides.

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