Chapter 0. What is Linear Algebra?

A good part of MA1S11 (and of MA1S12) is going to be about linear algebra. Here is an attempt to explain in advance what it is all about.

Unlike Mathematics in general, linear algebra is a rather completely understood subject. That is not to say that there are not problems in linear algebra that are interesting for research (there are!) but most of the basic ideas are well understood both from a theoretical and a practical point of view. The practical point of view means that there are known ways to solve most problems. In fact there are often several ways and what is important for a scientist is that there are efficient methods for large problems. For example, physicists, geneticists and chemists have problems that require these efficient methods.

From another point of view, very many problems of interest to scientists translate into nonlinear mathematical problems. Most of these cannot be solved in any nice way. However, an important technique for these intractable nonlinear problems is to find linear problems that approximate the nonlinear ones of interest. Linear algebra is then useful because there is usually a good way to solve the linear problem. But the difficulty then is to try to relate the solution of the approximate problem to the solutions of the actual (nonlinear) problem.

None of this says what linear algebra is. Perhaps as we learn the beginnings of the subject, we will start to grasp what it is about. One thing is the meaning of the word 'linear', which we will also see explained properly later.

It might be worth saying that squares, cubes, reciprocals, or products of variables (or unknowns in equations) will not occur in linear algebra. Well, they will not occur until you get to determinants and eigenvalues [which come in Hilary term, in MA1S12] when we do encounter some of them.

R. Timoney September 23, 2012