2E1 (Timoney) Tutorial sheet 5
[Tutorials November 8 – 9, 2006]

Name: Solutions

1. Find the partial derivatives with respect to x and y evaluated at $(x_0, y_0) = (2, -2)$ for

$$f(x,y) = \frac{x\cos(\pi y)}{x^2 + y^2}$$

Solution:

$$\begin{aligned}
\frac{\partial f}{\partial x} &= \frac{\cos(\pi y)(x^2+y^2) - x\cos(\pi y)(2x)}{(x^2+y^2)^2} \\
&= \frac{(x^2+y^2-2x^2)\cos(\pi y)}{(x^2+y^2)^2} \\
&= \frac{(y^2-x^2)\cos(\pi y)}{(x^2+y^2)^2} \\
\frac{\partial f}{\partial y} &= \frac{-\pi x\sin(\pi y)(x^2+y^2) - x\cos(\pi y)(2y)}{(x^2+y^2)^2} \\
&= \frac{-\pi x(x^2+y^2)\sin(\pi y) - 2xy\cos(\pi y)}{(x^2+y^2)^2} \\
\frac{\partial f}{\partial x}\,|_{(2,-2)} &= 0 \\
\frac{\partial f}{\partial y}\,|_{(2,-2)} &= \frac{0+8}{8^2} = \frac{1}{8}
\end{aligned}$$

2. Find the directional derivatives $D_{\mathbf{u}}f(x_0, y_0)$ for the same function $f(x, y)$ as in the previous problem, the same point (x_0, y_0), and $\mathbf{u} = (u_1, u_2)$ any unit vector.

   Solution: We know

$$D_{\mathbf{u}}f = \frac{\partial f}{\partial x}u_1 + \frac{\partial f}{\partial y}u_2$$

   and so, evaluating at $(x_0, y_0) = (2, -2)$

$$D_{\mathbf{u}}f(x_0, y_0) = D_{\mathbf{u}}f(2, -2) = 0u_1 + \frac{1}{8}u_2 = \frac{u_2}{8}$$

3. For the same $f(x, y)$, the same (x_0, y_0) and the corresponding point (x_0, y_0, z_0) on the graph $z = f(x, y)$, find parametric equations for the line which is tangent to the graph at (x_0, y_0, z_0) and perpendicular to the x-axis.

Solution: The line will be tangent to the curve where the graph $z = f(x, y)$ intersects the plane $x = x_0$, and so tangent to the curve

$$\begin{aligned} x &= x_0 \\ y &= y_0 + t \\ z &= f(x_0, y_0 + t) \end{aligned}$$

at $t = 0$. The tangent vector to that curve is

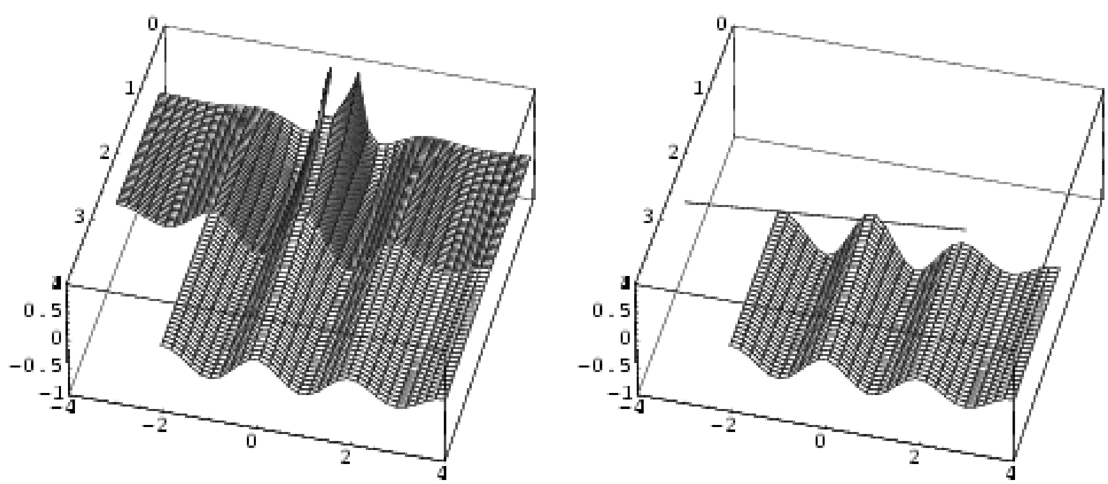
$$\left(0, 1, \frac{\partial f}{\partial y} \mid_{(x_0, y_0)}\right) = (0, 1, 1/8).$$

So the line is

$$\begin{aligned} x &= x_0 \\ y &= y_0 + t \\ z &= z_0 + t(1/16) \end{aligned}$$

and $z_0 = f(x_0, y_0) = \frac{2(1)}{4+4} = 1/4$. Thus the answer is

$$\begin{aligned} x &= 2 \\ y &= -2 + t \\ z &= \frac{1}{4} + \frac{t}{8}. \end{aligned}$$



The left hand graph shows $z = f(x, y)$ cut away at $(2, -2)$ (the x scale runs 0 to 4, and the y from -4 to 4). The right graph shows a part of $z = f(x, y)$ with the tangent line we computed.

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