2E1 (Timoney) Tutorial sheet 4
[Tutorials November 1 – 2, 2006]

Name: Solutions

1. Find cartesian equations for the line

$$\begin{aligned} x_1 &= 3 + 2t \\ x_2 &= 3 - 2t \\ x_3 &= 1 + 3t. \end{aligned}$$

Solution: Solving each of the 3 equatiosn for t we get

$$t = \frac{x_1 - 3}{2}, \quad t = \frac{x_2 - 3}{-2}, \quad t = \frac{x_3 - 1}{3}$$

and the caretesian equations are

$$\frac{x_1 - 3}{2} = \frac{x_2 - 3}{-2} = \frac{x_3 - 1}{3}$$

(One casn use (x, y, z) in place of (x_1, x_2, x_3) for the corrdinates.)

2. Find parametric equations for the line of intersection of the two planes

$$\begin{array}{rcrcrcl} 2x & - & 3y & + & 4z & = & 0 \\ -x & + & y & + & 2z & = & 3 \end{array}$$

Solution: The normal vectors to the planes are $(2, -3, 4)$ and $(-1, 1, 2)$. Thus the line is perpendicular to both vectors, or parallel to their cross product.

$$\begin{aligned} (2, -3, 4) \times (-1, 1, 2) &= \det \begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -3 & 4 \\ -1 & 1 & 2 \end{pmatrix} \\ &= \mathbf{i} \det \begin{pmatrix} -3 & 4 \\ 1 & 2 \end{pmatrix} - \mathbf{j} \det \begin{pmatrix} 2 & 4 \\ -1 & 2 \end{pmatrix} + \mathbf{k} \det \begin{pmatrix} 2 & -3 \\ -1 & 1 \end{pmatrix} \\ &= -10\mathbf{i} - 8\mathbf{j} - \mathbf{k} \\ &= (-10, -8, -1) \end{aligned}$$

To get a point on the line we can try for a point with $z = 0$ and this means solving the system of equations

$$\begin{array}{rcrcl} 2x & - & 3y & = & 0 \\ -x & + & y & = & 3 \end{array}$$

We see $y = 2x/3$ from the first equation and then the second says $-x + 2x/3 = 3$, or $-x/3 = 3$ or $x = -9$. And then $y = -6$. So the point $(9, 6, 0)$ is on the line of intersection.

Armed with a point $\mathbf{a} = (-9, -6, 0)$ and a vector $\mathbf{b} = (-10, -8, -1)$ parallel to the line, we can write parametric equations:

$$\begin{aligned} x &= -9 - 10t \\ y &= -6 - 8t \\ z &= 0 - t. \end{aligned}$$

3. Find parametric equations for the line in $\mathbb{R}^3$ that is perpendicular to the plane

$$-5x - y + z = 11$$

and contains the point $(1, 0, 2)$.

Solution: A vector normal to the plane is $(-5, -1, 1)$ and, since we know the point $(1, 0, 2)$ we can write down the equations

$$\begin{aligned} x &= 1 - 5t \\ y &= 0 - t \\ z &= 2 + t. \end{aligned}$$