2E1 (Timoney) Tutorial sheet 3
[Tutorials October 25 – 26, 2006]

Name: Solutions

1. Find the equation of the plane through $(1,2,5)$ perpendicular to the line

$$\begin{aligned} x_1 &= 3+2t \\ x_2 &= 3-2t \\ x_3 &= 1. \end{aligned}$$

Solution: A vector parallel to the line is $(2,-2,0)$ and this is then normal to the plane. So the plane has equation $2x_1 - 2x_2 = c$ (or $2x - 2y = c$ if you prefer) for some constant c.

As $(x_1, x_2, x_3) = (1,2,5)$ is on the plane we have to have $2 - 4 = c$ or $c = -2$ and the equation is $2x_1 - 2x_2 = -2$ (or $x_1 - x_2 = -1$).

2. Find the equation of the plane in $\mathbb{R}^3$ that contains the 3 points $(1,2,3)$, $(4,5,6)$ and $(1,5,7)$.

Solution: The differences

$$(4,5,6) - (1,2,3) = (3,3,3) \text{ and } (1,5,7) - (1,2,3) = (0,3,4)$$

are in the plane (or parallel to the plane). So the cross product of these two vectors is normal to the plane

$$\begin{aligned} (3,3,3) \times (0,3,4) &= \det \begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 3 & 3 \\ 0 & 3 & 4 \end{pmatrix} \\ &= \mathbf{i} \det \begin{pmatrix} 3 & 3 \\ 3 & 4 \end{pmatrix} - \mathbf{j} \det \begin{pmatrix} 3 & 3 \\ 0 & 4 \end{pmatrix} + \mathbf{k} \det \begin{pmatrix} 3 & 3 \\ 0 & 3 \end{pmatrix} \\ &= 3\mathbf{i} - 12\mathbf{j} + 9\mathbf{k} \\ &= (3,-12,9) = 3(1,-4,3) \end{aligned}$$

So the plane will have an equation of the form $x_1 - 4x_2 + 3x_3 = c$ (or $x - 4y + 3z = c$) for some c. Plugging in one of the three points we find c. Taking $(1,2,3)$ we get $1 - 8 + 9 = c$ or $c = 2$. So the equation is $x_1 - 4x_2 + 3x_3 = 2$.

(It is reassuring the check that the other two points $(4,5,6)$ and $(1,5,7)$ also satisfy this equation.)

3. Give an example of a function $f(x,y)$ so that its graph lies on the sphere $(x-2)^2 + (y-1)^2 + (z+3)^2 = 2$ (of radius $\sqrt{2}$ about $(2,1,-3)$). Give also the domain for your example function $f(x,y)$.

Solution: The points on the graph $z = f(x, y)$ must satify $(x-2)^2+(y-1)^2+(z+3)^2 = 2$, and we can go about solving this equation for z in terms of $f(x, y)$. We get

$$\begin{aligned} (z+3)^2 &= 2-(x-2)^2-(y-1)^2 \\ z+3 &= \pm\sqrt{2-(x-2)^2-(y-1)^2} \\ z &= -3\pm\sqrt{2-(x-2)^2-(y-1)^2} \end{aligned}$$

Since a function can only have one value at any point of its domain, we could choose, say,

$$f(x, y) = -3+\sqrt{2-(x-2)^2-(y-1)^2}$$

and this is ok as long as we are taking the square root of a nonnegative quantity.

So we restrict (x, y) to satisfy $(x-2)^2 + (y-1)^2 \leq 2$, which is the circle of radius $\sqrt{2}$ and center $(2, 1)$ in the plane togther with the inside of the circle. We could take this set as the domain of $f(x, y)$ and then we have a good definition of a function f.

Richard M. Timoney