# RC Filter Networks Lab Report

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**Abstract**

The aims of this experiment are to examine RC circuits as low-pass and high-pass filters to source voltages of sinusoidal and square wave signals. Their responses to different frequencies are investigated and the 'half power point' is measured.

## 1 Theory

In an RC circuit the total impedance is

$$Z = R - iX_C$$
$$Z = |Z| \exp(i\theta)$$

where $X_C = 1/\omega C$ and $\theta = \arctan(-X_C/R)$.

The voltage drop across the resistor and capacitor are given respectively by

$$V_R = \frac{R}{R - iX_C} V_{in}$$
$$V_C = \frac{-iX_C}{R - iX_C} V_{in}$$

### 1.1 High-pass

In a high-pass RC filter circuit the output voltage is taken across the resistor. From the equation above the magnitude of the output voltage is

$$\frac{|V_{out}|}{|V_{in}|} = \frac{R}{\sqrt{R^2 + X_C^2}}$$
$$\frac{|V_{out}|}{|V_{in}|} = \frac{R}{\sqrt{R^2 + (1/\omega C)^2}}$$
$$\frac{|V_{out}|}{|V_{in}|} = \frac{\omega C R}{\sqrt{1 + \omega^2 C^2 R^2}}$$

so $|V_{out}|$ increases with increasing $\omega$ to a maximum of $|V_{in}|$. It allows high frequencies through and attenuates low frequencies. Clearly it is correctly called a high-pass circuit.

The phase difference between the input voltage and the output voltage is (from the equation above)

$$\theta_{high} = \arctan\left(\frac{X_C}{R}\right)$$
$$\theta_{high} = \arctan\left(\frac{1}{\omega CR}\right)$$

so the output voltage leads the input voltage by $\theta_{high}$ and this phase difference decreases with increasing frequency.

## 1.2 Low-pass

In a low-pass circuit the output voltage is taken across the capacitor so the magnitude of the output voltage is in this case

$$\frac{|V_{out}|}{|V_{in}|} = \frac{X_C}{\sqrt{R^2 + X_C^2}}$$
$$\frac{|V_{out}|}{|V_{in}|} = \frac{1/\omega C}{\sqrt{R^2 + (1/\omega C)^2}}$$
$$\frac{|V_{out}|}{|V_{in}|} = \frac{1}{\sqrt{1 + \omega^2 C^2 R^2}}$$

clearly $|V_{out}|$ decreases with increasing $\omega$ so it attenuates high frequencies and passes low frequencies.

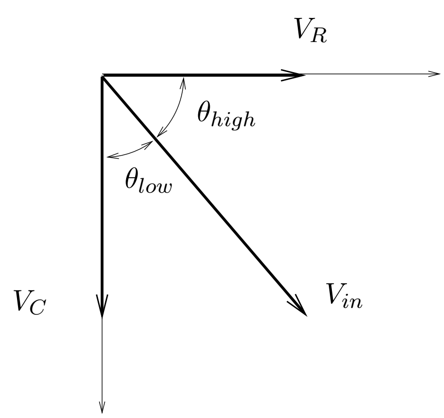


Figure 1: Phasor diagram showing the voltage drop across the capacitor, the resistor, and the source voltage. $V_C = IX_C$ and $V_R = IR$ so clearly $\tan(\theta_{high}) = X_C/R$ and $\theta_{low} = \theta_{high} - \frac{\pi}{2}$.

The phase difference between the input voltage and the output voltage is

$$\theta_{low} = \arctan\left(\frac{-R}{X_C}\right)$$
$$\theta_{low} = \arctan\left(-\omega CR\right)$$

the output voltage lags the input voltage by $\theta_{low}$ and increases with increasing frequency to a maximum of $\frac{\pi}{2}$.

These results can easily be summarized in a phasor diagram, see figure 1.

### 1.3 Integrating and differentiating circuits

Using Kirchoff's laws we can show that the voltage across a capacitor in an RC circuit with a constant voltage source $V_0$ is

$$V_C = V_0\left[1 - \exp(-t/\tau)\right]$$

where $\tau = RC$ is the time constant of the circuit. If $t = \tau$ then

$$\frac{V_C}{V_0} = 1 - \exp(-1)$$
$$\frac{V_C}{V_0} \approx 0.63$$

For a general source voltage $V_{in}$ the sum of $V_R = RI$ and $V_C = Q/C$ is $V_{in}$. Since $I = \mathrm{d}Q/\mathrm{d}t$ we have

$$V_R = RI$$
$$V_R = R\frac{\mathrm{d}(V_C C)}{\mathrm{d}t}$$
$$V_R = RC\frac{\mathrm{d}V_C}{\mathrm{d}t}$$

and similarly for $V_C$

$$V_C = \frac{1}{RC}\int V_R \mathrm{d}t$$

If we apply a square wave to this circuit with period $T$ then we have two distinct cases. If $T \gg RC$ then the capacitor charges fully every period and $V_C \approx V_{in}$, also

$$V_R \propto \frac{\mathrm{d}V_C}{\mathrm{d}t}$$
$$V_R \propto \frac{\mathrm{d}V_{in}}{\mathrm{d}t}$$

If $T \ll RC$ then the capacitor does not have time to charge and so $V_R \approx V_{in}$ and so

$$V_C \propto \int V_{in} \mathrm{d}t$$

In this way we have simple integrating and differentiating circuits.

# 2 High-Pass Filter

## 2.1 Method and Results

The high-pass RC filter circuit was set up as shown in figure 2. The peak-to-peak values of the output voltage, $V_{out}$, and the time difference, $t$, in seconds was measured for different frequencies and a constant $V_{in}$. The phase difference was calculated using $\theta_{high} = 2\pi f t$. Since the data would be plotted on a logarithmic scale the frequency was varied according to a logarithmic scale.

| $f$/Hz | $V_{out}$/V | $t$/ms | $\theta_{high}$/rad |
|---|---|---|---|
| 10±.02 | .11±.001 | 25±2 | 1.6±.1 |
| 14±.04 | .15±.001 | 17±1 | 1.50±.09 |
| 19±.03 | .20±.002 | 12±1 | 1.4±.1 |
| 26±.04 | .274±.002 | 9.5±.5 | 1.55±.08 |
| 36±.04 | .376±.002 | 6.5±.5 | 1.5±.1 |
| 49±.05 | .512±.004 | 5±.2 | 1.54±.06 |
| 68±.1 | .704±.004 | 3.4±.2 | 1.45±.09 |
| 94±.2 | .976±.004 | 2.5±.1 | 1.48±.06 |
| 130±.1 | 1.34±.01 | 1.8±.1 | 1.47±.06 |
| 178±.4 | 1.86±.02 | 1.25±.05 | 1.40±.06 |
| 245±.3 | 2.52±.01 | .85±.05 | 1.31±.08 |
| 337±.4 | 3.38±.02 | .6±.05 | 1.3±.1 |
| 464±.5 | 4.48±.04 | .4±.02 | 1.17±.06 |
| 640±.6 | 5.72±.04 | .25±.01 | 1.01±.01 |
| 880±2 | 7.04±.04 | .16±.01 | .88±.06 |
| 1210±5 | 8.32±.01 | .09±.005 | .68±.04 |
| 1670±10 | 9.60±.01 | .05±.005 | .52±.06 |
| 2300±20 | 10.1±.01 | .028±.002 | .40±.03 |
| 3200±10 | 10.4±.1 | .015±.001 | .30±.02 |
| 4500±10 | 10.7±.1 | .008±.0005 | .23±.01 |
| 6000±20 | 10.8±.1 | .0045±.0005 | .17±.02 |

$V_{in} = 11.1 \pm .05$V

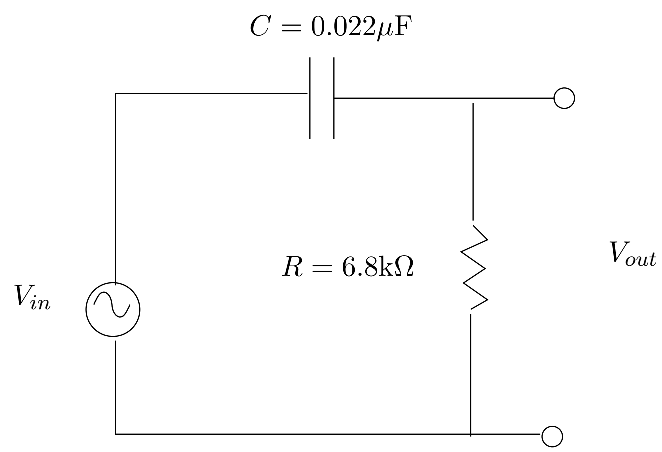


Figure 2: High-pass RC circuit. The output is taken over the resistor.

### 2.2 Discussion

Figure 3 shows the dependence of $V_{out}$ on the frequency of the input sinusoidal voltage, $f$. The error bars have been left out because they were too small to see. It is convenient to use a logarithmic scale on the x-axis because it allows us to look at a wide range of $f$ while keeping the numerical values low. This is an increasing function, tending towards zero. This trend matches with the theory for the high-pass circuit since $V_{out}$ tends to $V_{in}$ for higher frequencies.

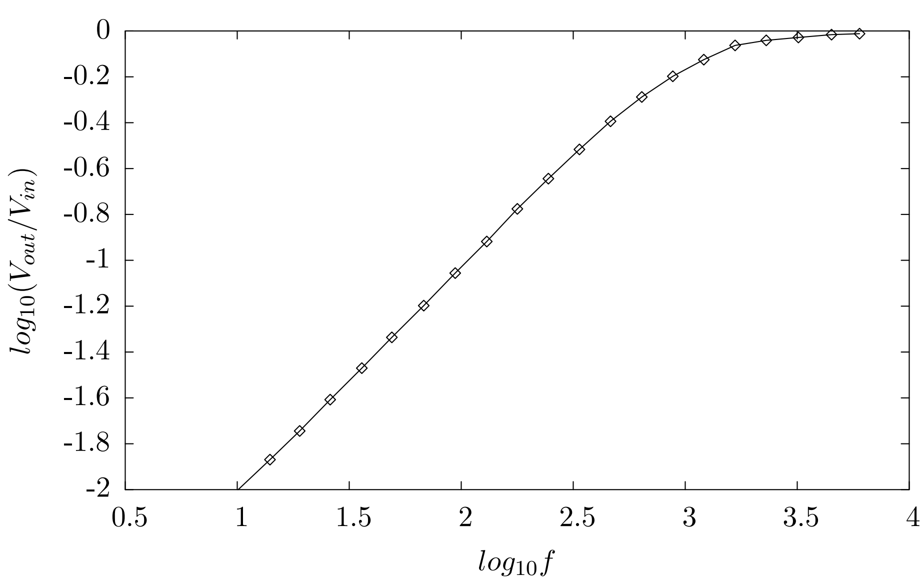


Figure 3: The voltage increases with increasing frequency for a high-pass circuit tending towards $V_{in}$ which would be zero on this graph.

The half power point is the point at which the output power is a half the input power or since $P \propto V^2$ the point at which the output voltage is $1/\sqrt{2}$ times the input voltage. This is called the 3 dB point because using the decibel scale

$$\begin{aligned} I &= 10\text{dB}\log_{10}(P_{in}) - 10\text{dB}\log_{10}(P_{out}) \\ &= 10\text{dB}\left(\log_{10}(P_{in}) - \log_{10}(P_{in}/2)\right) \\ &= 10\text{dB}\log_{10}(2) \\ &\approx 3\text{dB} \end{aligned}$$

We can work out the frequency at the half power point from the theory.

$$\begin{aligned} \frac{1}{\sqrt{2}} &= \frac{\omega CR}{\sqrt{1+\omega^2C^2R^2}} \\ \omega CR &= 1 \\ f &= \frac{1}{2\pi CR} \\ f &= \frac{1}{2\pi \times 0.022 \times 10^{-6} \times 6.8 \times 10^3} \\ f &= 1064\text{Hz} \end{aligned}$$

The half power point on the graph was found to be $\log_{10}(f) = 3.04 \pm 0.01$ so $f = 1100 \pm 30$Hz. so the measured value for the half power point agrees well with the theoretical value.

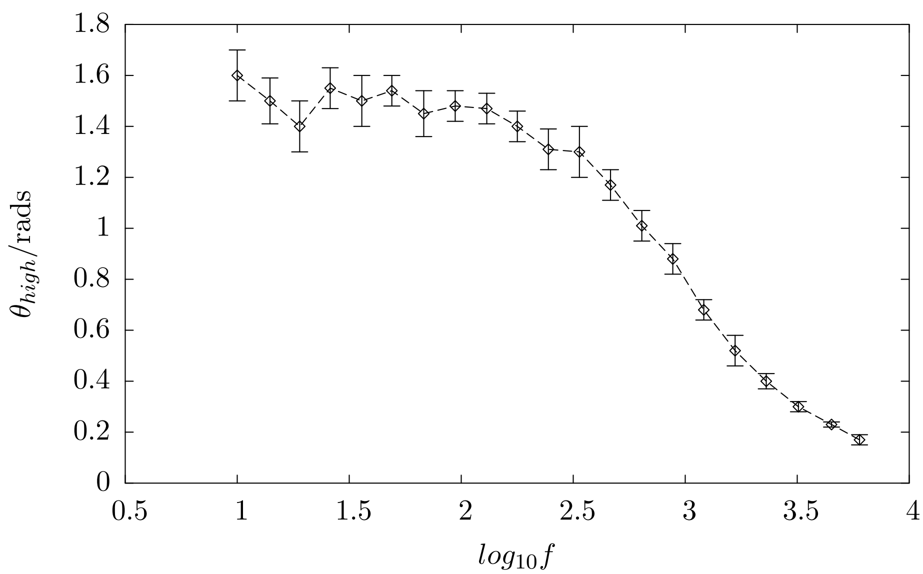


Figure 4: The phase difference in a high-pass circuit is a decreasing function tending towards zero.

Figure 4 shows the variation of the phase angle $\theta_{high}$ with $f$. Again the error bars on the x-axis were left out because they were small. The phase angle at the half power point was determined from the graph to be $\theta_{high} = .74 \pm .05$. At the half power point we have

$$\omega CR = 1$$

so

$$\begin{aligned}\theta_{high} &= \arctan(1)\\ \theta_{high} &= .785\end{aligned}$$

the measured phase angle at half power point is within experimental of the theoretical value. The input was leading the output throughout the high-pass experiment.

# 3 Low-pass Filter

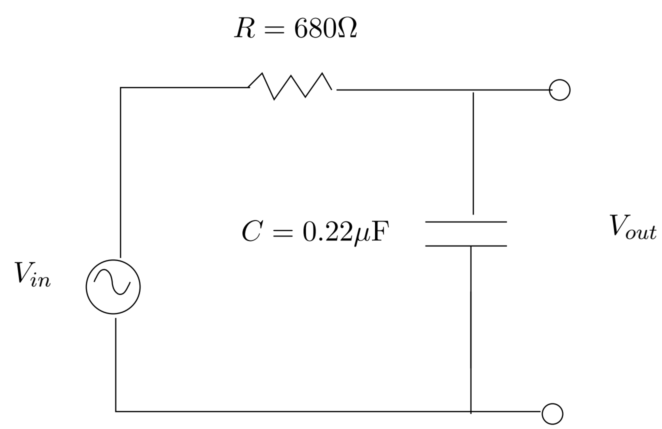


Figure 5: Low-pass RC circuit. The output is taken over the capacitor.

The low-pass RC filter circuit was set up as shown in figure 5. The frequency and phase shift at the half-power point were measured.

The frequency at the half-power point was $1.14 \pm .03$kHz. This is not within experimental error of the previously calculated theoretical value of 1064Hz. It is an error of approximately 6%. The time difference was $110 \pm 5\mu$s which corresponds to a phase difference of $\lambda_{low} = .79 \pm .06$ rad. This is within experimental error of the theoretical value of .785. In this case the output was leading the input voltage.

## 4 Square wave response of filter circuits

A square wave was applied to the high-pass and low-pass circuits shown in figures 2,5 respectively.

For the period $T \gg \tau$, the time constant, the output voltage shows exponential growth. The time constant was measured by checking the time at which the output voltage was 0.63 times the input voltage. This was found to be $151 \pm 2\mu$s. This is within experimental error of the calculated value of

$$\tau_{th} = CR = 0.22 \times 10^{-6} \times 680 = 149\mu\text{s}$$

The figures 6-8 show the three cases of $T \gg \tau$, $T \approx 2\tau$, and $T \ll \tau$ for the high-pass and low-pass circuits.

Figure 6(a) shows the high-pass circuit for $T = 7.15$ms. In this case $T \gg RC$ so the circuit acts as a differentiating circuit. If we compare figure 6(a) with figure 6(b) we see that the high-pass circuit shows the slope of the output from the low-pass circuit. Also we note that the output voltage from the low-pass circuit shown in figure 6(b) approximates the input square voltage.

Figures 7(a) and 7(b) show the case for $T \approx 2RC$. We can see the exponential growth and decay in figure 7(b) and the corresponding output over the resistor in figure 7(a). Again we note that the high-pass circuit displays the slope of the low-pass circuit and conversely the low-pass circuit shows the area under the high-pass circuit.

Figures 8(a) and 8(b) shows when $T = 30.40\mu$s so $T \ll RC$. The output across the high-pass filter approximates the square wave input voltage and so the output of the low-pass filter is approximately the integral of the square wave input. This is the integrating circuit.

These results can be qualitatively predicted if we know that the Fourier series of the square wave input is

$$x(t) = \sin(\omega t) + \frac{1}{3}\sin(3\omega t) + \frac{1}{5}\sin(5\omega t) + \dots$$

For $T \gg RC$ i.e. a low frequency input the high-pass filter will only allow through the high-frequency parts of the Fourier series, which gives us the wave shown in figure 6(a). The low-pass filter circuit on the other hand will allow through most of the series and so the output will approximate the input, as seen in figure 6(b).

If $T \ll RC$ the high-pass filter allows through all the components of the Fourier series and so the high-pass output approximates the input square wave. The low-pass filter circuit in this case attenuates the higher frequencies greatly and the lower frequencies that pass produce the wave shown in figure 8(b).

## 5 Conclusions

The half power point of the high-pass filter was found to be $f = 1100 \pm 30$Hz and the the output was lagging the input at this frequency by $\theta_{high} = .74 \pm .05$. These are both within experimental error of the theoretical values.

The half power point of the low-pass filter circuit was measured to be $f = 1140 \pm 30$Hz which is an error of 6% compared to the theoretical value of 1064Hz. The output was leading the input by $\theta_{low} = .79 \pm .06$rads.

The time constant of the low-pass circuit was measured to be $\tau = 151 \pm 2\mu$s. This is within experimental error of 149$\mu$s. The RC filter circuits were also investigated as simple integrating and differentiating circuits.

## References

[1] W.J. Duffin. *Electricity and Magnetism*. McGraw-Hill.