

UNIVERSITY OF DUBLIN

XMA11221

TRINITY COLLEGE

FACULTY OF ENGINEERING, MATHEMATICS
AND SCIENCE

SCHOOL OF MATHEMATICS

JF Maths, TP, TSM

Trinity Term 2011

MA1121 CONCEPTS OF ANALYSIS — MA1122 ANALYSIS ON THE REAL LINE

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Attempt all questions. All questions are weighted equally.
Log tables are available from the invigilators, if required.

MA1121 — Concepts of analysis

1. Show that the set $A = \left\{ \frac{2n+3}{n+4} : n \in \mathbb{N} \right\}$ has a minimum but no maximum.
2. Suppose that f is continuous with $f(0) < 1$. Show that there exists some $\delta > 0$ such that $f(x) < 1$ for all $-\delta < x < \delta$.
3. Show that the function f defined by

$$f(x) = \left\{ \begin{array}{ll} 2x + 1 & \text{if } x \leq 1 \\ x + 3 & \text{if } x > 1 \end{array} \right\}$$

is not continuous at $y = 1$.

4. Suppose that f is continuous with $f(x) \in \mathbb{Q}$ for all $x \in \mathbb{R}$. Show that f is constant.
5. Give an example of a bounded function which does not have a maximum on $[0, 1]$.

MA1122 — Analysis on the real line

6. Let $p > 0$ be fixed. Show that $\frac{\log x}{x^p} \leq \frac{1}{pe}$ for all $x > 0$.
7. Suppose that f is decreasing on $[0, 1]$. Show that f is integrable on $[0, 1]$.
8. Show that there exists a unique function f , defined for all $x \in \mathbb{R}$, such that

$$f'(x) = e^{-x^2}, \quad f(0) = 0.$$

Moreover, show that f is odd, namely that $f(-x) = -f(x)$ for all $x \in \mathbb{R}$.

9. Define a sequence $\{a_n\}$ by letting $a_1 = 1$ and

$$a_{n+1} = 3 + \sqrt{a_n} \quad \text{for each } n \geq 1.$$

Show that $1 \leq a_n \leq a_{n+1} \leq 9$ for each $n \geq 1$ and find the limit of this sequence.

10. Test each of the following series for convergence:

$$\sum_{n=1}^{\infty} \sin \frac{1}{n}, \quad \sum_{n=1}^{\infty} \cos \frac{1}{n}, \quad \sum_{n=1}^{\infty} \frac{(-1)^n e^{1/n}}{n}.$$

11. Suppose that the power series

$$f(x) = \sum_{n=0}^{\infty} a_n x^n$$

converges when $x = 2$. Show that it converges absolutely when $|x| < 2$.