

UNIVERSITY OF DUBLIN

XMA11221

TRINITY COLLEGE

FACULTY OF ENGINEERING, MATHEMATICS
AND SCIENCE

SCHOOL OF MATHEMATICS

JF Maths, TP, TSM

Trinity Term 2010

MA1121 CONCEPTS OF ANALYSIS & MA1122 ANALYSIS ON THE REAL LINE

Monday, May 10

Sports Centre

9:30 – 12:30

Dr. P. Karageorgis & Dr. J. Stalker

Attempt all questions. All questions are weighted equally.
Log tables are available from the invigilators, if required.

Section MA1121 — Concepts of analysis

1. Show that the set $A = \{\frac{n+1}{n+2} : n \in \mathbb{N}\}$ is such that $\sup A = 1$.
2. Show that f is continuous at all points when f is the function defined by

$$f(x) = \left\{ \begin{array}{ll} 3x - 1 & \text{if } x \leq 2 \\ 9 - 2x & \text{if } x > 2 \end{array} \right\}.$$

3. Suppose that f is continuous with $f(0) < 1$. Show that there exists some $\delta > 0$ such that $f(x) < 1$ for all $-\delta < x < \delta$. Hint: use the ε - δ definition for some suitable ε .
4. Show that $4x^4 + 22x^2 \geq 4x^3 - 11$ for all $x \in \mathbb{R}$. Hint: you need to find a min.

Section MA1122 — Analysis on the real line

5. Suppose that f is differentiable on $[a, b]$ and that $f'(x) > 0$ for all $a \leq x \leq b$. Show that if $f(a) \leq y \leq f(b)$, then there is exactly one $x \in [a, b]$ such that $f(x) = y$.

6. Prove or give a counterexample to the following statements:

(a) If f is differentiable and strictly increasing on (a, b) , then $f'(x) > 0$ for all $a < x < b$.

(b) If f is convex on (a, b) , then $f''(x) \geq 0$ for all $a < x < b$.

7. Prove that there is a function s , defined on all of \mathbb{R} , such that

$$s(0) = 0, \quad s'(x) = (1 + x^4)^{-1/2}.$$

Show that s is bounded.

8. Prove that the following series diverge:

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{1+n^2}}, \quad \sum_{n=2}^{\infty} \frac{1}{n \log n}, \quad \sum_{n=1}^{\infty} \sin n.$$