Revision Tutorial 1S2 JF Natural Sciences

1. Given three vectors $\vec{u} = (-3,\, 1,\, 2),\ \vec{v} = (4,\, 0,\, -8),\ \vec{w} = (6,\, -1,\, -4)$, obtain the components of the following combinations of vectors

   (a) $\vec{v} - \vec{w}$

   (b) $6\vec{v} + 2\,\vec{w}$

   (c) $-\vec{v} + \vec{u}$

   (d) $5(\vec{v} - 4\,\vec{u})$

   Find scalar quantities a, b, and c, such that $a\,\vec{u} + b\,\vec{v} + c\,\vec{w} = (2,\, 0,\, 4)$.

2. Evaluate the following given the three-by-three matrices

$$D = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}, \qquad E = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}.$$

   (a) $D - E$,

   (b) $4E - 2D$.

   (c) $-3(D + 2\,E)$,

   (d) tr (D) ,

   (e) tr $(D - 3E)$.

   (f) $D^T - E^T$,

   (g) $2E^T - 3D^T$.

3. Find the eigenvalues of the following matrix

$$A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}.$$

Problem 1 relates to Q6 and Q8 of Exercise set 3.1 on page 126, problem 2 relates to Q3 and Q4 of Exercise set 1.3 on page 35, and problem 3 refers to Q4(a) and Q5(a) of Exercise set 7.1 on page 362 of *Elementary Linear Algebra & Applications* by Howard Anton and Chris Rorres (7th edition Wiley). For similar problems and answers provided at the back see corresponding sections in the 8th edition, and in *Contemporary Linear Algebra* by Howard Anton and Robert C. Busby (Wiley) and also in *Linear Algebra and its Applications* by David C. Lay (Addison-Wesley).

BACKGROUND MATERIAL FOR EIGENVALUES OF A MATRIX

The eigenvalues of the following square matrix

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 10 & 0 & 0 \\ 13 & 1 & 2 \end{bmatrix}$$

are obtained from the condition that the defining equation for an eigenvalue λ

$$A\,V = \lambda\,V$$

or

$$(A - \lambda\,I)\,V = 0$$

has non trivial solutions for the vector V, namely the determinantal condition

$$\det(A - \lambda\,I) = 0$$

or

$$\begin{vmatrix} -\lambda & 0 & 1 \\ 10 & -\lambda & 0 \\ 13 & 1 & 2-\lambda \end{vmatrix} = 0$$

or

$$-\lambda\,[\,-\lambda(2-\lambda) - 0(1)\,] - 0\,[\,10(2-\lambda) - 0(13)\,] + 1[\,10(1) - (-\lambda)(13)\,] = 0$$

and therefore the characteristic equation, polynomial in λ, of the matrix A reads

$$\lambda^3 - 2\,\lambda^2 - 13\,\lambda - 10 = 0$$

upon a change of sign. The product of the roots of the cubic equation is 10 here. Using the divisors of 10 with either sign, trial substitution in the cubic leads to

$$\lambda = +5\,, \qquad \lambda' = -1\,, \qquad \lambda'' = -2\,,$$

the eigenvalues of the matrix A. Should they be required, the eigenvectors of A corresponding to each eigenvalue may be determined from

$$A\,V = \lambda\,V\,, \qquad A\,V' = \lambda'\,V'\,, \qquad A\,V'' = \lambda''\,V''\,.$$

See the appropriate sections of *Elementary Linear Algebra & Applications* by Howard Anton and Chris Rorres (7th edition Wiley), corresponding sections in the 8th edition, in *Contemporary Linear Algebra* by Howard Anton and Robert C. Busby (Wiley) and in *Linear Algebra and its Applications* by David C. Lay (Addison-Wesley) for many problems similar to those in all the lectures and tutorials together with answers given at the end of each book to provide guidance in attempting questions.