SCHOOL OF MATHEMATICS TRINITY COLLEGE DUBLIN

Matrices Tutorial 1S2 JF Natural Sciences

Consider the following three by three matrix M with matrix elements shown

$$M = \begin{bmatrix} 6 & 1 & 1 \\ 3 & 7 & 6 \\ 1 & 2 & 3 \end{bmatrix}.$$

1. Write down the matrix $0.1\,M$ with its matrix entries.
2. Find the sum of each of the three columns of the matrix M.
3. Calculate the matrix sum $M + M$.
4. Display the transpose of the matrix M, namely M^T.
5. Evaluate the trace of the matrix M, namely tr (M).
6. Exhibit the zero 3×3 matrix and the unit 3×3 matrix I.
7. Obtain the product of the matrix M with the unit matrix I.
8. Find the product of the matrix M with the column vector

$$\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}.$$

9. Determine the product of the row vector [1 1 1] with the matrix M.
10. Obtain the product of the matrix M and its transpose M^T.
11. Is the product of matrices $M\,M^T$ a symmetric matrix?
12. Calculate the product of the matrices M^T and M.
13. Is the product $M^T M$ a a symmetric matrix ?
14. Does the product $M\,M^T$ equal the product $M^T M$?

Background Material for the Tutorial on Matrix Algebra

- The transpose of the following three by three matrix

$$\begin{bmatrix} 7 & 1 & 1 \\ 1 & 6 & 1 \\ 2 & 3 & 8 \end{bmatrix}$$

  is the 3×3 matrix

$$\begin{bmatrix} 7 & 1 & 2 \\ 1 & 6 & 3 \\ 1 & 1 & 8 \end{bmatrix}.$$

- The trace of the following three by three matrix is: $5 + 6 + 7 = 18$

$$\begin{bmatrix} 5 & 1 & 2 \\ 1 & 6 & 1 \\ 4 & 3 & 7 \end{bmatrix}.$$

- The product of the following two matrices

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \times \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & k \end{bmatrix}$$

  is the three by three matrix

$$\begin{bmatrix} 1a + 2b + 3c & 1d + 2e + 3f & 1g + 2h + 3k \\ 4a + 5b + 6c & 4d + 5e + 6f & 4g + 5h + 6k \\ 7a + 8b + 9c & 7d + 8e + 9f & 7g + 8h + 9k \end{bmatrix}.$$

  Observe that the matrix elements in the product are the result of taking a dot product of a row vector in the first matrix with a column vector in the second matirx.

- The transpose of the second matrix in the product is, in fact,

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix}.$$