

Linear Equations Tutorial 1S2

JF Natural Sciences

1. Is the number of solutions of the following system of two linear equations (a) zero? (b) one? (c) infinite? Find the solution set of the equations.

$$2x - 3y = 14$$

$$6x - 9y = 42$$

2. Is the number of solutions of the following system of two linear equations (a) zero? (b) one? (c) infinite? Obtain the solution set of the equations.

$$4x - 5y = 15$$

$$6x - 9y = 21$$

3. Is the number of solutions of the following system of two linear equations (a) zero? (b) one? (c) infinite? Write down the solution set of the equations.

$$3x - 4y = 16$$

$$6x - 8y = 22$$

4. Apply the *Gauss elimination* procedure to arrange zeros below an echelon of leading ones for the augmented matrix that is associated with the linear system of three equations

$$2x - 6y + 8z = 22$$

$$4x - 9y + 7z = 23$$

$$5x + 3y - 2z = 25.$$

5. Use the *Jordan elimination* procedure to arrange zeros above the leading ones for the augmented matrix resulting from the *Gauss elimination* of the same linear system of three equations
6. Hence determine from the final augmented matrix values of the three quantities x , y and z .
7. Verify that the values of x , y and z obtained do satisfy all three equations of the above linear system.

A Matrix Question. A sample analysis shows a molar composition of 21 for copper, 16 for iron, and 31 for sulphur. Minerals contributing to the overall copper, iron, sulphur content are Chalcopyrite (Cu_5FeS_2), Troilite (FeS), and Bornite (Cu_5FeS_4). Find the amounts in the sample of Chalcopyrite (x), Troilite (y), and Bornite (z).

A study of the amounts of copper, iron and sulphur in the three minerals yields

$$\begin{aligned}x &+ 5z = 21 && (\text{Cu}) \\x + y + z &= 16 && (\text{Fe}) \\2x + y + 4z &= 31 && (\text{S}).\end{aligned}$$

The *augmented* matrix that represents this linear system of equations is

$$\begin{bmatrix} 1 & 0 & 5 & 21 \\ 1 & 1 & 1 & 16 \\ 2 & 1 & 4 & 31 \end{bmatrix}$$

Arrange *zeros* below the leading *one* in the first row by (a) replacing the second row with the result of subtracting the first row from the second row, (b) replacing the third row with the result of subtracting twice the first row from the third row

$$\begin{bmatrix} 1 & 0 & 5 & 21 \\ 0 & 1 & -4 & -5 \\ 0 & 1 & -6 & -11 \end{bmatrix}$$

Arrange a *zero* below the leading *one* in the second row by replacing the third row with the result of subtracting the second row from the third row

$$\begin{bmatrix} 1 & 0 & 5 & 21 \\ 0 & 1 & -4 & -5 \\ 0 & 0 & -2 & -6 \end{bmatrix}$$

When the third row is divided by -2 the procedure of *Gaussian elimination* has resulted in an augmented matrix with leading *ones* in the non-zero rows

$$\begin{bmatrix} 1 & 0 & 5 & 21 \\ 0 & 1 & -4 & -5 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

To complete *Gauss-Jordan elimination* procedure with *zeros* above each leading *one* (a) replace the second row with the result of adding four times the third row to the second row, (b) replace the first row with the result of subtracting five times the third row from the first row

$$\begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

A *zero* already exists above the leading *one* in the second row so that a subtraction using the second row is not necessary here. Writing the equations explicitly with variables x , y and z yields

$$\begin{aligned}x + 0y + 0z &= 6 \\0x + 1y + 0z &= 7 \\0x + 0y + 1z &= 3\end{aligned}$$

whence the solution set is $x = 6$, $y = 7$, $z = 3$. This is readily checked by substitution.