Matrix Algebra 1S2 Answers JF Natural Sciences

1. The matrix M and the matrix $0.1\,M$ with their matrix entries are

$$\begin{bmatrix} 6 & 1 & 1 \\ 3 & 7 & 6 \\ 1 & 2 & 3 \end{bmatrix}, \qquad \begin{bmatrix} 0.6 & 0.1 & 0.1 \\ 0.3 & 0.7 & 0.6 \\ 0.1 & 0.2 & 0.3 \end{bmatrix}.$$

2. The sum of the matrix elements in the first column of the matrix M is 10 as is the sum of the matrix entries in the second and third columins of the matrix M. The column entries of $0.1\,M$ all sum to unity, an example of a probability matrix.

3. The matrix sum $M + M$ is

$$\begin{bmatrix} 12 & 2 & 2 \\ 6 & 14 & 12 \\ 2 & 4 & 6 \end{bmatrix}.$$

4. The transpose of the matrix M, namely M^T is the following (and note that the transpose of M^T is the matrix M)

$$\begin{bmatrix} 6 & 3 & 1 \\ 1 & 7 & 2 \\ 1 & 6 & 3 \end{bmatrix}.$$

5. The trace of the matrix M is the sum of its diagonal elements (so that it is also the trace of the transposed matrix M^T).

$$\text{tr}\,(M) \;=\; 6 + 7 + 3 \;=\; 16\,.$$

6. The zero 3×3 matrix and the unit 3×3 matrix I are

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

7. The product of the matrix M with the unit matrix I is

$$\begin{bmatrix} 6 & 1 & 1 \\ 3 & 7 & 6 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 1 & 1 \\ 3 & 7 & 6 \\ 1 & 2 & 3 \end{bmatrix}$$

being the result of a calculation involving dot products of a row from the first matrix and a column from the second matrix.

$$\begin{bmatrix} 6\times1+1\times0+1\times0 & 6\times0+1\times1+1\times0 & 6\times0+1\times0+1\times1 \\ 3\times1+7\times0+6\times0 & 3\times0+7\times1+6\times0 & 3\times0+7\times0+6\times1 \\ 1\times1+2\times0+3\times0 & 1\times0+2\times1+3\times0 & 1\times0+2\times0+3\times1 \end{bmatrix}.$$

8. The product of the matrix M with the transpose of the vector [1 3 1] is ten times the same transposed vector as indicated in the following matrix equation. (It is termed an eigenvector of the matrix M corresponding to the eigenvalue 10).

$$\begin{bmatrix} 6 & 1 & 1 \\ 3 & 7 & 6 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 30 \\ 10 \end{bmatrix} .$$

9. The product of the row vector [1 1 1] with the matrix M sums its columns thus

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 6 & 1 & 1 \\ 3 & 7 & 6 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 10 & 10 & 10 \end{bmatrix} .$$

10. The product of the matrix M and its transpose M^T is

$$\begin{bmatrix} 6 & 1 & 1 \\ 3 & 7 & 6 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 6 & 3 & 1 \\ 1 & 7 & 2 \\ 1 & 6 & 3 \end{bmatrix} = \begin{bmatrix} 38 & 31 & 11 \\ 31 & 94 & 35 \\ 11 & 35 & 14 \end{bmatrix}$$

being the result of a calculation involving dot products of a row from the first matrix and a column from the second matrix.

$$\begin{bmatrix} 36 + 1 + 1 & 18 + 7 + 6 & 6 + 2 + 3 \\ 18 + 7 + 6 & 9 + 49 + 36 & 3 + 14 + 18 \\ 6 + 2 + 3 & 3 + 14 + 18 & 1 + 4 + 9 \end{bmatrix}$$

11. The matrix product $MM^T = A$ is symmetric since the matrix elements a_{ij} in row i and column j of the matrix A satisfy $a_{ij} = a_{ji}$ for all $i, j = 1, 2, 3$.

12. The product of the transposed matrix M^T and M is

$$\begin{bmatrix} 6 & 3 & 1 \\ 1 & 7 & 2 \\ 1 & 6 & 3 \end{bmatrix} \begin{bmatrix} 6 & 1 & 1 \\ 3 & 7 & 6 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 38 & 29 & 27 \\ 29 & 94 & 49 \\ 27 & 49 & 14 \end{bmatrix}$$

being the result obtained from the intermediate calculation of

$$\begin{bmatrix} 36 + 1 + 1 & 6 + 21 + 2 & 6 + 18 + 3 \\ 6 + 21 + 2 & 9 + 49 + 36 & 1 + 42 + 6 \\ 6 + 18 + 3 & 1 + 42 + 6 & 1 + 4 + 9 \end{bmatrix}$$

13. The matrix product $M^TM = B$ is symmetric since the matrix elements b_{ij} in row i and column j of the matrix B satisfy $b_{ij} = b_{ji}$ for all $i, j = 1, 2, 3$.

14. The matrix product MM^T does not equal the product M^TM in general.