1. A factor three relates the two equations. The number of solutions of the equations $2x-3y=14$ and $6x-9y=42$ is infinite (c). The general solution may be written $2x=14+3t,\ y=t,$ where t has any value. Substituting $t=2u$ permits x to be written without fractions in the general solution, that is, $x=7+3u,\ y=2u.$ The solution set for $\{x,\,y\}$ is $\{7+3u,\,2u\}$, where u takes any value.

2. The number of solutions of the linear equations $4x-5y=15$ and $6x-9y=21$ is one (b). The solution set is $\{5,\,1\}$ for $\{x,\,y\}$.

3. The number of solutions of the linear equations $3x-4y=16$ and $6x-8y=22$ is zero (a). Twice the first equation is inconsistent with the second equation. The solution set is $\{\}$, the null set.

4. *Gaussian elimination* seeks to arrange zeros below an echelon of leading ones for the augmented matrix that is associated with the system of three equations

$$\begin{array}{rcrcrcr} 2x & - & 6y & + & 8z & = & 22 \\ 4x & - & 9y & + & 7z & = & 23 \\ 5x & + & 3y & - & 2z & = & 25 \end{array} \qquad \begin{bmatrix} 2 & -6 & 8 & 22 \\ 4 & -9 & 7 & 23 \\ 5 & 3 & -2 & 25 \end{bmatrix}.$$

Division of the first row by 2 arranges a leading *one* in the first row. Multiply the new first row by -4 to prepare for addition to the second row as indicated below. A zero appears in the second row below the leading 1 in the first row.

$$\begin{bmatrix} 1 & -3 & 4 & 11 \\ 4 & -9 & 7 & 23 \\ 5 & 3 & -2 & 25 \end{bmatrix} \qquad \begin{array}{r|rrrr} -4\times\texttt{1st Row} & -4 & 12 & -16 & -44 \\ \texttt{2nd Row} & 4 & -9 & 7 & 23 \\ \hline \texttt{New 2nd Row} & 0 & 3 & -9 & -21 \end{array}$$

Multiplying the first row by -5 prepares for a subsequent addition to the third row as the following shows. Addition achieves a zero in the third row below the leading *one* in the first row upon replacement by the new third row.

$$\begin{bmatrix} 1 & -3 & 4 & 11 \\ 0 & 3 & -9 & -21 \\ 5 & 3 & -2 & 25 \end{bmatrix} \qquad \begin{array}{r|rrrr} -5\times\texttt{1st Row} & -5 & 15 & -20 & -55 \\ \texttt{3rd Row} & 5 & 3 & -2 & 25 \\ \hline \texttt{New 3rd Row} & 0 & 18 & -22 & -30 \end{array}$$

In the following augmented matrix notice that the second row is divisible by 3 and that the third row is divisible by 2. Dividing the second row by 3 introduces a leading *one* as indicated. To simplify, divide the third row by 2.

$$\begin{bmatrix} 1 & -3 & 4 & 11 \\ 0 & 3 & -9 & -21 \\ 0 & 18 & -22 & -30 \end{bmatrix} \qquad \begin{array}{rrrrr} \texttt{2nd Row / 3 :} & 0 & 1 & -3 & -7 \\ \texttt{3rd Row / 2 :} & 0 & 9 & -11 & -15 \end{array}$$

Multiply the second row by -9 for addition to the third row as indicated. Addition gives a zero in the third row below the leading 1 in the second row.

$$\begin{bmatrix} 1 & -3 & 4 & 11 \\ 0 & 1 & -3 & -7 \\ 0 & 9 & -11 & -15 \end{bmatrix} \qquad \begin{array}{r|rrrr} -9\times\texttt{2nd Row} & 0 & -9 & 27 & 63 \\ \texttt{3rd Row} & 0 & 9 & -11 & -15 \\ \hline \texttt{New 3rd Row} & 0 & 0 & 16 & 48 \end{array}$$

In the following augmented matrix observe that 16 divides the third row. Dividing the third row by 16 introduces a leading *one* in the third row.

$$\begin{bmatrix} 1 & -3 & 4 & 11 \\ 0 & 1 & -3 & -7 \\ 0 & 0 & 16 & 48 \end{bmatrix} \qquad \texttt{3rd Row / 16 :} \quad 0 \quad 0 \quad 1 \quad 3$$

Gaussian elimination has produced an augmented matrix in *row echelon* form.

5. To continue to *reduced row echelon* form use multiples of the third row to seek *zeros* above the leading *one* in that third row. Multiply the third row by 3 for a subsequent addition to the second row before replacement of row two.

$$\begin{bmatrix} 1 & -3 & 4 & 11 \\ 0 & 1 & -3 & -7 \\ 0 & 0 & 1 & 3 \end{bmatrix} \qquad \begin{array}{rrrrr} \texttt{2nd Row} & 0 & 1 & -3 & -7 \\ 3 \times \texttt{3rd Row} & 0 & 0 & 3 & 9 \\ \hline \texttt{New 2nd Row} & 0 & 1 & 0 & 2 \end{array}$$

Multiply the third row by -4 in advance of addition to the first row as shown. Addition yields a *zero* in the first row above the leading 1 in the third row.

$$\begin{bmatrix} 1 & -3 & 4 & 11 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \qquad \begin{array}{rrrrr} \texttt{1st Row} & 1 & -3 & 4 & 11 \\ -4 \times \texttt{3rd Row} & 0 & 0 & -4 & -12 \\ \hline \texttt{New 1st Row} & 1 & -3 & 0 & -1 \end{array}$$

Multiply the second row by 3 for addition to the first row as shown below. Addition achieves a 0 in the first row above the leading 1 in the second row.

$$\begin{bmatrix} 1 & -3 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \qquad \begin{array}{rrrrr} \texttt{1st Row} & 1 & -3 & 0 & -1 \\ 3 \times \texttt{2nd Row} & 0 & 3 & 0 & 6 \\ \hline \texttt{New 1st Row} & 1 & 0 & 0 & 5 \end{array}$$

Gauss-Jordan elimination has provided the following augmented matrix in *reduced row echelon* form. The associated system of equations is exhibited.

$$\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \qquad \begin{array}{rcrcrcr} 1x & + & 0y & + & 0z & = & 5 \\ 0x & + & 1y & + & 0z & = & 2 \\ 0x & + & 0y & + & 1z & = & 3 \end{array}$$

6. The solution $x = 5$, $y = 2$, $z = 3$, may be evaluated from the above equations.

7. The solution can be checked by substitution in the system of linear equations

$$\begin{array}{rcrcrcrcrcrcr} 2 \times 5 & - & 6 \times 2 & + & 8 \times 3 & = & 10 & - & 12 & + & 24 & = & 22 \\ 4 \times 5 & - & 9 \times 2 & + & 7 \times 3 & = & 20 & - & 18 & + & 21 & = & 23 \\ 5 \times 5 & + & 3 \times 2 & - & 2 \times 3 & = & 25 & + & 6 & - & 6 & = & 25 \end{array}$$