Asymmetry $A_{\rm N}$ for Elastic Collisions of Helium-3 on Ions in the CNI Region

The single spin asymmetry $A_{\rm N}$ of a spin half fermion (proton, helion, antiproton, ...) of mass $m_{\rm f}$ and charge $Z_{\rm f}\, e$ scattering elastically off a positive ion of charge $Z_{\rm A}\, e$ (of any spin) may be estimated in the CNI region by using an approximate small angle helicity nonflip amplitude that ignores an energy factor s, a hadronic real part ratio ρ, and a Coulomb phase

$$\frac{Z_{\rm f}\, Z_{\rm A}\, \alpha}{t} \; + \; i\, \beta_{\rm L}\, \frac{\sigma_{\rm tot}({\rm fA})}{8\,\pi}$$

where $\sigma_{\rm tot}({\rm fA})$ refers to the total cross section for hadronic fermion ion scattering, α denotes the fine structure constant, and $\beta_{\rm L}$ is the laboratory velocity of the incident fermion—essentially unity at high energy. If the magnetic moment of the incident spin half fermion is $\mu_{\rm f}$ in units of the nuclear magneton involving proton mass $m_{\rm p}$, the approximate high energy fermion helicity flip amplitude, neglecting the hadronic part, is the electromagnetic helicity flip amplitude

$$\frac{Z_{\rm f}\, Z_{\rm A}\, \alpha}{t} \, \left(\frac{\mu_{\rm f}}{Z_{\rm f}} \, - \, \frac{m_{\rm p}}{m_{\rm f}} \right) \frac{\sqrt{-t}}{2\, m_{\rm p}} \, .$$

Noting that the ratio of electromagnetic flip to electromagnetic nonflip amplitudes is at most 3%, that is, 0.1% for squared amplitudes, the fermion asymmetry $A_{\rm N}$ has an extremum (for nuclear size effects $< 1/\sqrt{-t_{\rm ext}}$) at invariant momentum transfer close to

$$-t_{\rm ext} \; = \; \frac{8\pi\alpha}{\sigma_{\rm tot}({\rm fA})} \; |Z_{\rm f}\, Z_{\rm A}\,|\, \sqrt{3}$$

the extreme value of the incident particle spin asymmetry (either a maximum or minimum depending on the sign) at squared momentum transfer $-t_{\rm ext}$ being

$$A_{\rm N}^{\rm ext}({\rm fA}) \; = \; \frac{Z_{\rm f}}{|\, Z_{\rm f}\,|} \, \left(\frac{\mu_{\rm f}}{Z_{\rm f}} - \frac{m_{\rm p}}{m_{\rm f}} \right) \frac{\sqrt{-3\, t_{\rm ext}}}{4 m_{\rm p}} \, .$$

In the case of a nucleus A (p, 3He, or 12C, say) the ratio of the asymmetry extrema for helion nucleus and proton nucleus scattering is

$$\frac{A_{\rm N}^{\rm ext}({\rm hA})}{A_{\rm N}^{\rm ext}({\rm pA})} \; = \; \frac{\mu_{\rm h}/Z_{\rm h} - m_{\rm p}/m_{\rm h}}{\mu_{\rm p} - 1} \, \left(\frac{\sigma_{\rm tot}({\rm hA})/Z_{\rm h}}{\sigma_{\rm tot}({\rm pA})} \right)^{-1/2} \; = \; -0.780 \sqrt{\frac{2\sigma_{\rm tot}({\rm pA})}{\sigma_{\rm tot}({\rm hA})}} \, .$$

The masses and magnetic moments of proton and helium-3 nuclei (helion) are

$$m_{\rm h}/m_{\rm p} = 2.99315, \quad \mu_{\rm p} = 2.79285 \text{ and } \mu_{\rm h} = -2.1275 \text{ nuclear magnetons}$$

See W. W. MacKay, `http://www.rhichome.bnl.gov/AP/ap_notes/ap_note_296.pdf`

See N. H. Buttimore, Spin 2002 Brookhaven, editor Yousef I. Makdisi et al., page 844.

The electromagnetic current matrix element for a spin half fermion of mass m with initial and final four momenta p and p' respectively may be written in two ways

$$\bar{u}' \left(\gamma^\mu F_1 \,+\, \frac{1}{2} [\,\gamma^\mu, \gamma^\nu\,] \, \frac{p_\nu - p'_\nu}{2\,m} \, F_2 \right) u \,=\, \bar{u}' \left(\frac{p^\mu + p'^\mu}{2m} \, F_1 \,+\, \frac{1}{2} [\,\gamma^\mu, \gamma^\nu\,] \, \frac{p_\nu - p'_\nu}{2\,m} \, G_{\rm M} \right) u$$

where the electromagnetic form factors $F_1(t)$ and $G_{\rm M}(t)$, with $t = (p' - p)_\mu (p' - p)^\mu$, have static values related to the charge and magnetic moment of the fermion

$$F_1(0) \,=\, q\,, \qquad \frac{G_{\rm M}(0)}{2\,m} \,=\, \mu' = \mu \, \frac{e}{2\,m_p}\,,$$

noting here that the magnetic moment μ' is normally quoted as μ when given in terms of the nuclear magneton involving the unit charge e and the mass of a proton $m_{\rm p}$. Apart from a factor $(2\pi)^3$ for the current, the above analysis is based upon that given on page 454 of "Quantum Field Theory I" by Steven Weinberg with identifications

$$\gamma^\mu = i\gamma^\mu_{\rm W}\,, \qquad g^{\mu\nu} = -\eta^{\mu\nu}_{\rm W} = {\rm diag}\,(1,\, -1,\, -1,\, -1)\,,$$

$$F_1 = qF_1^{\rm W}, \qquad F_2 = -qG^{\rm W} = 2mqF_2^{\rm W}, \qquad G_{\rm M} = qF^{\rm W}.$$

Observe that the Dirac form factor $F_1(t)$ includes the charge q in its normalisation so that expressions also apply to the case of a neutral fermion such as the neutron. The following decomposition for the spinors $\bar{u}' = \bar{u}(p')$ and $u = u(p)$,

$$\bar{u}' \left\{ 2\,m\,\gamma^\mu \,-\, (p' + p)^\mu \,-\, \tfrac{1}{2} [\,\gamma^\mu\,,\,\gamma^\nu\,]\,(p - p')_\nu \right\} u \,=\, 0\,,$$

leads to a relation for the Sachs magnetic form factor of the spin half particle

$$G_{\rm M}(t) \,=\, F_1(t) + F_2(t)$$

so that the anomalous magnetic moment of the fermion of electric charge $q = Ze$ is

$$\frac{F_2(0)}{2\,m} \,=\, \mu' - \frac{q}{2m} \,=\, \frac{e}{2} \left(\frac{\mu}{m_p} \,-\, \frac{Z}{m} \right)\,.$$

Note that the magnetic moment μ' has the Dirac value $q/2m$ in the absence of an anomaly, a quantity that is equal to Zm_p/m when expressed in terms of the nuclear magneton $e/2m_p$. With $j = 1/2$ for the spin of a fermion, Weinberg's (10.6.22) and (10.6.23) provides $\mu^{\rm W}/j = qF^{\rm W}(0)/m$. With the normalization condition exhibited after (10.6.18), Weinberg's (10.6.17) indicates

$$2mF_2^{\rm W}(0) = F^{\rm W}(0) - F_1^{\rm W}(0) \,= 2m\mu^{\rm W}/q - 1\,; \qquad qF_2^{\rm W}(0) = \mu^{\rm W} - q/(2m)$$

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