A SPIN ON THE PROTON FOR SEEKING NUCLEON STRUCTURE

Nigel Buttimore

Trinity College Dublin

22 May 2009

DAMTP Cambridge

Cavendish Laboratory

OUTLINE

Introduction to spin and the spin structure of the nucleon

- Spin polarised protons and their primary structure functions
- Helicity contributions to a sum rule for the spin of a proton
  - Evaluating polarisation levels that have been achieved for nucleons
  - Asymmetry from spin dependence in diffractive elastic scattering
- Methods for providing an intense beam of polarised antiprotons

Conclusions and outlook for neutron and proton spin structure functions

INTRODUCTION TO SPIN

Mass and spin are fundamental properties of particles and composites.
Spin had an uncertain beginning until the Dirac equation of 1928.

There is a hint of spin in the quaternionic nabla introduced by Hamilton

$$\nabla \; = \; i\,\partial_1 \, + j\,\partial_2 \, + k\,\partial_3$$

Quaternions $i, \; j, \; k$ may be viewed as Pauli matrices divided by $\sqrt{-1}$ within the free Weyl equation as an example.

The total angular momentum for a composite particle at rest is its spin.

SPIN OF A NUCLEON

In the naïve quark model with three valence quarks

- The incident energy is much less than the constituent quark mass
- The spin of the nucleon is the sum of the constituent quark spins

In quantum chromodynamics with many partons

- The energy of scattering is much greater than the current quark mass
- The number of quarks and gluons is influenced by the probing energy

PROTON HELICTY IN QCD

A stationary proton comprising various partons has angular momentum

$$\sum_{\text{flavour}} \langle\, P,\, S_3 = \tfrac{1}{2} \mid J^3_{\rm f} \mid P,\, S_3 = \tfrac{1}{2}\,\rangle \quad = \quad \frac{1}{2}$$

with quark, antiquark, gluon and orbital angular momentum elements

- The ith component of a quark angular momentum operator is(Qiu 2009)

$$J^{\,i}_q \quad = \quad \int d^{\,3}x\; \psi^\dagger_q \left[\, \gamma^i \gamma_5 \;-\; i\, \epsilon^{ijk} x^j D^k \,\right] \psi_q$$

- The ith gluon angular momentum operator in QCD has element (Ji)

$$J_g^{\,i} \quad = \quad \int d^{\,3}x \; \epsilon^{ijk}\, \epsilon^{klm}\, T^{0l}\, x^m$$

with a cross product of a $\nu = 0$ momentum tensor $T^{\nu\lambda}$ and location x^μ

- Matrix elements of the partonic operators dominate at high energy

- A sum rule for the spin of a proton relates to quarks and gluons

$$\frac{1}{2}\Sigma \, + \, L_q \, + \, J_g \, + \, L_g \quad = \quad \frac{1}{2}$$

A positive helicity state nucleon is described by helicity PDFs at scale Q

$$\Delta f_j(x,Q^2) \;=\; f_j^{\rightarrow}(x,Q^2) \;-\; f_j^{\leftarrow}(x,Q^2)$$

involving partons of type j with positive $(\rightarrow)$ and negative $(\leftarrow)$ helicity

- The contribution of valence and sea quarks to the proton helicity is

$$\Delta\Sigma \quad = \quad \int_0^1 dx \sum_{\text{flavour}} \left[\, \Delta q_f(x) \;+\; \Delta\bar{q}_f(x) \,\right]$$

  the quark having light-cone momentum fraction x of the nucleon's

Such integrals may be found from measured double spin asymmetries

$$A_{\mathrm{LL}} \quad = \quad \frac{d\Delta\sigma}{d\sigma} \quad = \quad \frac{d\sigma_{\rightarrow}^{\rightarrow} \;\;-\;\; d\sigma_{\leftarrow}^{\rightarrow}}{d\sigma_{\rightarrow}^{\rightarrow} \;\;+\;\; d\sigma_{\leftarrow}^{\rightarrow}}$$

The polarised pp cross section difference would involve terms of the form

$$d\,\Delta\sigma \;=\; \int dx\,\Delta q(x) \int dx'\,\Delta g(x')\; d\,\Delta\hat{\sigma}_{qg} \;+\; \mathcal{O}(\alpha_S^2) \;+\; \mathcal{O}(p_T^{-1})$$

summing over all initial partons, with $d\,\Delta\hat{\sigma}_{qg}$ being a partonic cross section.

- A similar sum holds for the unpolarised cross section $d\sigma$ at large p_T
- An extra fragmentation function may appear for particular observables
- A NLO analysis using $d\,\Delta\hat{\sigma}_{qg}$ and scale evolution of PDFs is available

Just one parton distribution appears for deep inelastic scattering (DIS)

EMC, SMC, COMPASS, E1.., HERMES, HALL-A, CLAS, PHENIX, STAR

- Flavour distributions result from Semi-Inclusive DIS, $l\,N \;\rightarrow l\,h\,X$

A next to leading order QCD global analysis of asymmetry data reveals

$$\begin{aligned} \Delta u \;+\; \Delta\bar{u} &= 81\,\% \\ \Delta d \;+\; \Delta\bar{d} &= -46\,\% \\ \Delta g &= -\;8\,\% \end{aligned} \qquad\qquad \begin{aligned} \Delta\bar{u} &= 4\,\% \\ \Delta\bar{d} &= -11\,\% \\ \Delta\bar{s} &= -\;6\,\% \end{aligned}$$

Surprisingly, quarks and antiquarks only contribute 24 % to proton spin and gluons only $-8\,\%$

de Florian, Sassot, Stratmann, Vogelsang (Phys Rev Lett, 2008)

Attention now focuses on orbital angular momentum for quarks and gluons

TRANSVERSE MOTION OF A PARTON

Space-time symmetry can induce zero values for particular asymmetries

- Parity conserved interactions lead to the result: $A_{\rm L} = 0$ (longitudinal)
- Time reversal invariance for inclusive DIS implies: $A_{\rm N} = 0$ (transverse)

Observation of a nonzero single spin asymmetry needs a T-odd triple product

$$A_{\rm N} \quad \propto \quad i\,\varepsilon^{ijk}\,s_i\,p_j\,l_k \quad \propto \quad i\,\epsilon_{\kappa\lambda\mu\nu}\,s^\kappa\,l^\lambda\,p_a^\mu\,p_b^\nu$$

involving a sufficient number of vectors to select a unique scattering plane

s^κ, p_a^μ, p_b^ν being incoming spin and momenta and l^λ an outgoing momentum

Transversity is the spin flip density at leading twist (factorisation $\approx\ Q^{-2}$)

$$\delta q(x) \quad = \quad \Delta_{\mathrm{T}}\, q(x) \quad = \quad h_1(x) \quad = \quad q^{\uparrow}(x) - q^{\downarrow}(x)$$

involving quarks having momentum fraction x with transverse polarisation

- parallel ($\uparrow$) to that of the nucleon's transverse polarisation minus
- antiparallel ($\downarrow$) to the same transverse polarisation of the nucleon

The transversity distribution function is accessed via the double asymmetry

$$A_{\mathrm{TT}}^{p\,p} \quad = \quad \frac{d\delta\sigma}{d\sigma} \quad = \quad \frac{d\sigma(\uparrow\uparrow) \quad - \quad d\sigma(\uparrow\downarrow)}{d\sigma(\uparrow\uparrow) \quad + \quad d\sigma(\uparrow\downarrow)}$$

in the Drell-Yan production of lepton pairs from a quark antiquark pair

For colliding transversely polarised proton beams the double asymmetry is

$$A_{\rm TT}^{p\,p} = \frac{d\Delta\hat{\sigma}}{d\hat{\sigma}} \frac{\sum^{q} e_q^2 \left[h_{1\,q}^{\,p}(x_1)\; h_{1\,\bar{q}}^{\,p}(x_2) \;+\; h_{1\,\bar{q}}^{\,p}(x_1)\; h_{1\,q}^{\,p}(x_2) \right]}{\sum_{q} e_q^2 \left[q^{\,p}(x_1)\; \bar{q}^{\,p}(x_2) \;+\; \bar{q}^{\,p}(x_1)\; q^{\,p}(x_2) \right]}$$

The sum is dominated by the more highly charged “up” quarks at large x

$$A_{\rm TT}^{p\,p} \quad \approx \quad \frac{d\Delta\hat{\sigma}}{d\hat{\sigma}} \times \frac{h_{1\,u}^{\,p}(x_1)\; h_{1\,\bar{u}}^{\,p}(x_2) \;+\; h_{1\,\bar{u}}^{\,p}(x_1)\; h_{1\,u}^{\,p}(x_2)}{u^{\,p}(x_1)\; \bar{u}^{\,p}(x_2) \;+\; \bar{u}^{\,p}(x_1)\; u^{\,p}(x_2)}$$

- The small number of sea antiquarks in a proton diminishes the asymmetry
- A proton’s quarks reacting with an antiproton’s antiquarks enhances $A_{\rm TT}^{\bar{p}\,p}$

In a proton antiproton Drell-Yan process the analysing power is

$$A_{\mathrm{TT}}^{p\,\bar{p}} \quad \approx \quad \frac{d\Delta\hat{\sigma}}{d\hat{\sigma}}\,\frac{h_{1\,u}^{\,p}\,(x_1)\;h_{1\,\bar{u}}^{\,\bar{p}}\,(x_2)}{u^{\,p}\,(x_1)\;\bar{u}^{\,\bar{p}}\,(x_2)}$$

Valence partons are much more plentiful in the proton antiproton reaction

Anselmino, Barone, Drago, Nikolaev, Phys Lett 2004

- Consider the case where beams have polarisations P_1 and P_2 (not 100%)
- If the momentum fractions x_1 and x_2 are equal the measured A_{TT} is

$$A_{\mathrm{TT}}^{p\,\bar{p}} \quad \approx \quad P_1\,P_2\,\frac{d\Delta\hat{\sigma}}{d\hat{\sigma}}\,\left[\frac{h_{1\,u}^{\,p}\,(x_1)}{u^{\,p}\,(x_1)}\right]^2, \quad (x_1 = x_2)$$

Methods of measuring beam polarisations at high energy are required.

There is an urgent need also for an intense stored polarised antiproton beam

Hadronic polarisations tend to vanish at higher collision energies making the evaluation of the level of proton polarisation a challenging problem

A source of neutrons with oriented spin may be available for eRHIC in the form of polarized helium-3 nuclei which, being charged, are candidates for electromagnetic hadronic interference polarimetry

Trueman, AIP Conf Proc 980 (2008)

- Pomeron spin dependent couplings from spin asymmetries

Trueman, Phys Rev D77 (2008)

Polarimetry for helium-3 beams is discussed in tandem with proton beams

INCIDENT HADRON POLARISATION ?

The RHIC pp collider is opening up a new frontier in hadronic spin studies

- Its polarized beams provide a new range of tests of the Standard Model
- The detailed partonic structure of the nucleon and of ions will emerge

An interchanged particle reaction $p\,p\uparrow \rightarrow p\,p$ gives absolute p polarisation

- A hydrogen jet of known polarisation calibrates the proton polarisation
- A very thin carbon fixed target serves as a relative proton polarimeter

The relative polarimeter acts continuously having been calibrated by a jet

A polarimeter requires a process with nonvanishing high energy polarisation

- Spin one photon exchange suggests the Primakoff or a Coulomb effect
- Electromagnetic hadronic interference has proved more reliable in tests
- Proton ion scattering has about 4% asymmetry in the interference region

A spin half hadron of mass m, charge Ze, magnetic moment μ scattering elastically off a charge $Z'e$ has an asymmetry that involves an interference

$$-2\,\mathrm{Im}\left[\frac{Z\,Z'\,\alpha}{t}+i\,\frac{\sigma_{\mathrm{tot}}}{8\,\pi}\right]\left[\left(\frac{Z}{m}-\frac{\mu}{m_p}\right)\frac{Z'\,\alpha}{2\,\sqrt{-t}}+\text{hadronic spin-flip}\right]^*$$

of helicity nonflip & flip amplitudes with electromagnetic & hadronic parts

Including the spin averaged denominator, the asymmetry is proportional to

$$A_{\mathrm{N}} \propto \frac{\sqrt{x}}{x^2+3}, \qquad x = \frac{t_{\mathrm{opt}}}{t}, \qquad t_{\mathrm{opt}} = -\frac{8\sqrt{3}}{\sigma_{\mathrm{tot}}}\pi\alpha\,|Z\,Z'|$$

the optimum value of which occurs at $x = 1$, that is, at transfer $t = t_{\mathrm{opt}}$

- The optimum value varies slowly with squared energy s as $1/\sqrt{\sigma_{\mathrm{tot}}(s)}$
- It is either a maximum or minimum depending on the sign of a term in

$$A_{\mathrm{N}}^{\mathrm{opt}} = \frac{1}{4Z}\left(\frac{\mu}{m_{\mathrm{p}}} - \frac{Z}{m}\right)\sqrt{-3\,t_{\mathrm{opt}}}$$

Neglected are hadronic helicity flip amplitudes and two photon exchange

The ratio of the asymmetry extrema for polarised helions and protons is

$$\frac{A_{\rm h}^{\rm opt}}{A_{\rm p}^{\rm opt}} \quad = \quad \frac{\mu_{\rm h}/Z - m_{\rm p}/m_{\rm h}}{\mu_{\rm p} - 1} \sqrt{\frac{Z\sigma_{\rm tot}^{\rm p}}{\sigma_{\rm tot}^{\rm h}}} \quad = \quad -\,0.6366 \sqrt{\frac{3\,\sigma_{\rm tot}^{\rm p}}{\sigma_{\rm tot}^{\rm h}}}$$

when a spin half proton or helion scatters elastically on a carbon ribbon

The mass ratio & magnetic moments of the proton and helium-3 nuclei are

$$m_{\rm h}/m_{\rm p} = 2.99315, \qquad \mu_{\rm p} = 2.79285, \qquad \mu_{\rm h} = -2.1275$$

so the negative magnetic moment of the helion indicates that the optimum asymmetry corresponds to a minimum in contrast to the proton's maximum

A more complete study includes discussion of the hadronic nonflip real part, helicity flip amplitudes and a two photon exchange phase

NB, Kopeliovich, Leader, Soffer, Trueman, Phys Rev D59 (1999)

ANOMALOUS MAGNETIC MOMENT

A study of the electromagnetic current matrix element leads to the factor

$$\left(\frac{\mu}{m_{\rm p}} - \frac{Z}{m}\right)$$

for a fermion of mass m and charge $q = Ze$ with initial and final p_μ, p'_μ

$$\bar{u}' \left\{ (p' + p)^\mu F_1 - \tfrac{1}{2} [\gamma^\mu, \gamma^\nu] (p' - p)_\nu G_{\rm M} \right\} u / 2m$$

where the electromagnetic form factors $F_1(t)$ and $G_{\rm M}(t)/(2m)$ with

$$t = (p' - p)_\mu (p' - p)^\mu,$$

have static values equal to the charge and magnetic moment of the fermion

$$F_1(0) \, = \, q \, , \qquad \frac{G_{\rm M}(0)}{2\,m} \, = \mu' = \mu \, \frac{e}{2\,m_{\rm p}}$$

noting that μ' is normally quoted as μ in nuclear magnetons.

An alternative expression for the current uses $F_1(t)$ and $F_2(t)$

$$\bar{u}' \, \left\{ \gamma^\mu \, F_1 \, - \, \tfrac{1}{2} \, [\, \gamma^\mu , \, \gamma^\nu \,] \, (p' - p)_\nu \, F_2 \right\} \, u$$

in a normalisation where the electromagnetic form factors are related by

$$G_{\rm M}(t) \, = \, F_1(t) \, + \, 2mF_2(t)$$

$$G_{\rm E}(t) \, = \, 2mF_1(t) \, + \, tF_2(t)$$

so that a fermion with charge $q = Ze$ has anomalous magnetic moment

$$F_2(0) \; = \; \mu' - \frac{q}{2m} = \frac{e}{2}\left(\frac{\mu}{m_p} - \frac{Z}{m}\right) \, .$$

The Dirac magnetic moment is $q/2m$, or Zm_p/m , in nuclear magnetons.

One photon exchange amplitudes are useful for helion proton polarimetry.

If the velocities of particles A and B in the centre of momentum frame are

$$\beta_{\rm A} = \left(1 + m_{\rm A}^2/k^2\right)^{-1/2} \qquad \beta_{\rm B} = \left(1 + m_{\rm B}^2/k^2\right)^{-1/2}$$

and the momentum of each particle in that particular frame is given by

$$4k^2 = s - 2m_{\rm A}^2 - 2m_{\rm B}^2 + \left(m_{\rm A}^2 - m_{\rm B}^2\right)^2/s$$

The one photon amplitudes for the scattering of particles A and B are

$$\phi_1^\gamma + \phi_3^\gamma = -G_{\mathrm{M}}^{\mathrm{A}}\, G_{\mathrm{M}}^{\mathrm{B}} + \left(4 + \frac{t}{k^2}\right)\left(m_{\mathrm{A}} m_{\mathrm{B}}\, F_2^{\mathrm{A}} F_2^{\mathrm{B}} + \frac{s - m_{\mathrm{A}}^2 - m_{\mathrm{B}}^2}{2\,t}\, F_1^{\mathrm{A}} F_1^{\mathrm{B}}\right)$$

$$\phi_1^\gamma - \phi_3^\gamma = -G_{\mathrm{M}}^{\mathrm{A}}\, G_{\mathrm{M}}^{\mathrm{B}}\,,$$

$$\phi_2^\gamma - \phi_4^\gamma = \frac{s^2 - \left(m_{\mathrm{A}}^2 - m_{\mathrm{B}}^2\right)^2}{4\,s}\left(4 + \frac{t}{k^2}\right) F_2^{\mathrm{A}} F_2^{\mathrm{B}} + \left(\frac{m_{\mathrm{A}}}{k} F_1^{\mathrm{A}} - 2k F_2^{\mathrm{A}}\right) \times \left(\frac{m_{\mathrm{B}}}{k} F_1^{\mathrm{B}} - 2k F_2^{\mathrm{B}}\right)$$

$$\phi_2^\gamma + \phi_4^\gamma = 0\,,$$

$$\phi_5^\gamma = +\sqrt{-\frac{1}{t} - \frac{1}{4k^2}}\left(\frac{m_{\mathrm{B}}}{\beta_{\mathrm{A}}} F_1^{\mathrm{A}} F_1^{\mathrm{B}} - 2k\sqrt{s}\, F_1^{\mathrm{A}} F_2^{\mathrm{B}} + \frac{m_{\mathrm{A}} t}{\beta_{\mathrm{B}}} F_2^{\mathrm{A}} F_2^{\mathrm{B}}\right)$$

$$\phi_6^\gamma = -\sqrt{-\frac{1}{t} - \frac{1}{4k^2}}\left(\frac{m_{\mathrm{A}}}{\beta_{\mathrm{B}}} F_1^{\mathrm{A}} F_1^{\mathrm{B}} - 2k\sqrt{s}\, F_1^{\mathrm{B}} F_2^{\mathrm{A}} + \frac{m_{\mathrm{B}} t}{\beta_{\mathrm{A}}} F_2^{\mathrm{A}} F_2^{\mathrm{B}}\right)$$

The differential cross section is (O'Brien & NB, Czech J Phys, 2006)

$$\frac{d\sigma}{dt} = \frac{\pi}{k^2 s}\left[\frac{1}{4\,t^2}\left(G_{\rm E}^{\rm A}\,G_{\rm E}^{\rm B}\right)^2 + \frac{1}{2}\left(G_{\rm M}^{\rm A}\,G_{\rm M}^{\rm B}\right)^2 + \right.$$

$$\left. + \frac{\left(m_{\rm A}^2 - m_{\rm B}^2\right)^2 - su}{t^2}\left(\frac{{G_{\rm E}^{\rm A}}^2 - t\,{G_{\rm M}^{\rm A}}^2}{4\,m_{\rm A}^2 - t}\right)\left(\frac{{G_{\rm E}^{\rm B}}^2 - t\,{G_{\rm M}^{\rm B}}^2}{4\,m_{\rm B}^2 - t}\right)\right]$$

a generalisation of the Rosenbluth formula for lepton proton collisions

where the expression above results from a spin sum of helicity amplitudes

$$\frac{d\sigma}{dt} = \frac{\pi}{2k^2 s}\left(|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2 + 2\,|\phi_5|^2 + 2\,|\phi_6|^2\right)\ .$$

CONCLUSIONS

The spin structure of the nucleon probes QCD deeply in interesting ways

- Spin structure functions of the proton are steadily emerging
- Helicity contributions to the proton spin sum rule are surfacing
  - Evaluating proton polarisation levels is continuously improving
  - Polarized helium-3 elastic and neutron levels may be forthcoming
- Providing an intense beam of polarised antiprotons is challenging

The outlook is good for neutron and proton spin structure functions