Overview of Processes Involved in Spin Transfer Collisions

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Abstract. An outline is provided of methods for polarizing antiprotons that include the possible use of channelling in a bent crystal and also the technique of scattering off leptons or protons at a suitably small angle. In the method of channelling it is suggested that the angular dependence of the analysing power of the incident particle is cubic in the angle for single scattering in the crystal when this takes place within the region of electromagnetic hadronic interference as is most likely for channelled particles. Polarization transfer in the scattering of antiprotons off leptons or protons is discussed in addition where emphasis is laid on the angular integrals over spin observables appropriate for spin filtering.

Keywords: antiproton, polarization, channelling, spin asymmetry, analysing power **PACS:** 13.88.+e, 14.20.Dh, 13.85.Dz, 14.60.Cd, 13.60.-r

INTRODUCTION

Seeking the partonic structure of the nucleon has been a preoccupation now for some considerable time; probing its spin degrees of freedom has revealed a number of surprises over the years. There is a distinct need for studies of the nucleon using antiquarks as probes and, in particular, spin polarized antiprotons for transversity studies [1].

An obvious source for such partons is a beam of antiprotons and the challenge is to polarize them in sufficient numbers to be adequate for the spin measurements of interest when endeavouring to understand the complete spin structure of a hadron. At the Bodega Bay Workshop on Polarized Antiprotons held during April 1985 in California the following ideas were highlighted [2]

- Decay in flight of produced antihyperons
- Spin filtering: polarized hydrogen target in a storage ring
- · Stochastic techniques à la stochastic cooling
- Dynamic polarization: polarized electrons and microwaves
- Spontaneous spin-flip synchrotron radiation
- Spin-flip synchrotron radiation induced by an X-ray laser
- · Directly produced polarized antiprotons by scattering
- Repeated Stern-Gerlach deflection through quadrupoles
- Formation of antihydrogen: use of atomic beam methods
- Polarization during storage in a Penning trap
- Polarizing by channelling through a thin metallic foil
- · Interaction with polarized photons from a diamond crystal

We consider here channelling in a bent crystal as a method of inducing polarization and also discuss fermion fermion elastic collisions as a way to build up polarization through polarization transfer from a polarized target in a scattering process.

CHANNELLING

Bent crystals have channelled many types of particle at a large number of accelerators including, for example, CERN[3], RHIC[4], IHEP[5], and Fermilab[6]. It has been suggested that such crystals may polarize fermions, particularly antiprotons, through repeated interaction with the nuclei of a curved channel in the lattice [7]. Measurements performed with many particle types over a large range of energies confirm the expression for the critical angle of the axial channelling effect in crystals derived by Lindhard,

$$\psi = \sqrt{\frac{4Ze^2}{pvd}} \tag{1}$$

where p is the laboratory momentum of the incident particle with velocity v and unit charge (of either sign) while Ze is the charge of the lattice nuclei and d the interatomic spacing for the particular crystal axis under investigation. The above formula described very well the angular behaviour of channelling in the early experiments where the angular half-widths at half maximum $\Delta \psi$ were proportional to ψ with a constant of proportionality close to 1.0 for positively charged positrons and protons and about 0.6 for negatively charged electrons [8]. A somewhat similar factor would be expected to apply in the case of negatively charged antiprotons.

To examine the level of polarization to be expected when a hadronic fermion glances off one of the charged nuclei of a string of atoms forming the channel, consider the magnitude of the momentum transferred, q, in scattering at a small laboratory angle θ

$$q = p \theta \tag{2}$$

If, for the moment, we study collisions at the same angle θ along the row of nuclei, the maximum momentum transfer at a given incident momentum *p* involves the greatest bending angle between successive atoms in the crystal, which for silicon is approximately $\theta_{\text{max}} = 50$ picoradians. The momentum transferred, however, must not be so large as to induce an escape from the channel via an excessive centrifugal force [3], a difficulty that now needs addressing.

The radius of curvature, r, of the bent crystal corresponding to an interatomic length d is about $r = d/\theta$, again for a small angle of bend. An incident fermion following the curved path responds to a centrifugal force of magnitude

$$\frac{pv}{r} = \frac{pv\theta}{d} = \frac{qv}{d}$$
(3)

that does not vary substantially with transverse location in the channel. The potential difference between one plane and the next, distant d away, is therefore qv and this should

be less than the potential difference U_0 between the centre of a channel and its edge

$$qv < U_0 \tag{4}$$

where, for silicon, $U_0 \approx 20$ eV. We shall see that inducing polarization requires as large a momentum transfer as is available. The above analysis reveals that, for incident momenta bounded by about p < 400 GeV/c, bending the crystal to its greatest extent provides a modicum of momentum transfer that does not, however, lead to de-channelling.

On the other hand, for incident momenta above about p > 400 GeV/c, the crystal bend should be relaxed to ensure that the momentum transfer is limited by $q < U_0$ in order to avoid loss of particles from the channel due to the centripetal acceleration that haunts the process.

Asymmetry

To understand the development of the level of polarization arising from the asymmetric scattering imposed by a bent crystal we observe that, in terms of the Sommerfeld fine structure constant α , electromagnetic and hadronic effects contribute about equally to the elastic collisions of hadrons of velocity $v = \beta c$ and unit charge off nuclei of charge Ze at a critical momentum transfer of [9]

$$q_c = \sqrt{\frac{8\pi Z\alpha}{\beta\sigma_{\rm tot}}} \tag{5}$$

where the total cross section here relates to the scattering of protons or antiprotons off a nucleus in the crystal lattice. As $q_c \approx 20 \text{ MeV}/c$ for collisions involving silicon with Z = 14 in the Coulomb interference region, the scattering within a channel referred to above would occur deep inside the interference region and low q approximations may be conducted accordingly.

It has been suggested that such scattering of fermions, p or \bar{p} , may polarize them [7]. To study the effect, consider the interference of helicty flip and helicty nonflip amplitudes in the case of elastic collisions of protons with mass m and anomalous magnetic moment κ on the spinless isotopes of a target nucleus, very similar to that for lattice nuclei with other spins [10]

$$M_+ \propto p \,\sigma_{\rm tot} \left(i + \rho - \frac{q_c^2}{q^2} \right)$$
 (6)

$$M_{-} \propto p \,\sigma_{\rm tot} \left(iI + R - \frac{1}{2} \beta \kappa \frac{q_c^2}{q^2} \right) \frac{q}{m}$$
 (7)

where the constant of proportionality is independent of energy and largely independent of momentum transfer also as hadronic and electromagnetic form factors have similar *t* dependencies at the small *t* values of interest, dependencies that cancel in the expression for the analyzing power. High energy approximations are appropriate for the amplitudes. Quantities ρ and R are the real parts of hadronic helicty nonflip and flip amplitudes respectively as a proportion of the imaginary hadronic nonflip part and are neglected in the following analysis at the large laboratory momenta suggested above. The corresponding imaginary hadronic helicty flip part is denoted I.

In the interference region of momentum transfer, $0.009 < q^2 < 0.041$ (GeV/c)², measurements of the analyzing power for proton carbon elastic scattering indicate that at 21.7 GeV/c [11]

$$R = 0.088 \pm 0.058$$
 and $I = -0.161 \pm 0.226$ (8)

and, similarly, in the region of momentum transfer $0.001 < q^2 < 0.032$ single spin asymmetries for proton proton elastic scattering at laboratory momenta 24 and 100 GeV/*c* provide similar constraints that restrict the sizes of hadronic helicity-flip amplitudes at these energies [12]. A report on an analysis relating to the polarized proton programme at RHIC involving the elastic scattering of protons on protons in collider or fixed target mode and of proton beams on carbon targets provides a recent summary of the spin dependence of high energy hadron elastic scattering [13].

Analyzing power

An extremum, maximum or minimum, of the single spin asymmetry $A_{\rm N}$ of amount

$$A_{\rm N}^{\rm max} = \frac{\kappa - 2I}{4m} \sqrt{3} q_{\rm m} \tag{9}$$

occurs near $q_{\rm m} = 3^{1/4} q_{\rm c}$, where κ is the anomalous magnetic moment of the hadron. For particle channelling below the region of electromagnetic hadronic interference [14] $q \ll q_{\rm c}$, the analyzing power normal to the scattering plane behaves as a cubic in the momentum transfer variable q

$$A_{\rm N} \approx \frac{\kappa - 2I}{m} \frac{q^3}{q_{\rm c}^2} \,. \tag{10}$$

At lower energies of scattering, an additional term, inversely proportional to the square of the momentum, would appear with the anomalous moment κ in the above expression [15]. Independently of the energy of the incident particle, however, the low value of the analyzing power A_N resulting from q being bounded by $U_0/v \approx 20 \text{ eV}/c$ while $q_c \approx 20 \text{ MeV}/c$ a million times greater, appears to indicate that the level of polarization achievable, even after passage close to the approximately hundred million nuclei of a 5 cm silicon crystal for example, would be insufficient for the purposes of possible forthcoming investigations.

An important assumption underlies the method of axial channelling, namely that the motion of the incident particle between rows of nuclei in the crystal may be described by a plane wave, thus permitting the interaction to be treated as a normal two particle collision process.

Far beyond the small values of momentum transfer where channelling occurs and outside the electromagnetic hadronic interference region, $q \gg q_c$, the analysing power has a linear dependence on the momentum transfer variable q

$$A_{\rm N} \approx \frac{2\rho I - 2R}{m} q \tag{11}$$

as expected in a region of hadronic dominance. The detailed dynamics of the eventual build-up of polarization may have to take account of the singular terms in q arising from photon exchange [16] appearing in the appropriate double spin asymmetries A_{ij} , K_{ij} , and D_{ij} and such concerns are addressed in the next section.

To summarize, keeping antiprotons in a channel of width d between atomic planes, the potential corresponding to the centrifugal force pv/r, the relativistic version of mv^2/r , must not exceed the known Si potential difference of about 20 eV between mid channel and the row of screened Z = 14 charges on the silicon atoms a distance d/2 away. Successive collisions of channelled fermions in a crystal may amplify their polarization before losses due to wide angle scattering diminish the luminosity of the beam. The analyzing power has an extremum in the interference region, in general a maximum for protons and a minimum for antiprotons.

The behaviour of the asymmetry is reasonably well understood below the interference region as it only depends upon a known imaginary part of the spin averaged hadronic amplitude and calculable spin dependent electromagnetic amplitudes, modulated slightly by helicity flip hadronic amplitudes that measurements appear to indicate are comparatively unimportant. The resulting polarization seems to be insufficient for the requirements of studies relating to transversity [1] and to the evaluation of time-like electromagnetic form factors [17]. The inclusion of the rôle of spin correlation, transfer and depolarization observables is unlikely to enhance substantially the limited polarization achievable after many nuclei have been traversed.

POLARIZATION TRANSFER

Selective attenuation of particles circulating in a storage ring due to their spin dependent hadronic or electromagnetic interaction with a polarized target was suggested by Csonka in 1968 as a way of polarizing a beam [18]. Such a method has been employed [19] to generate a beam of polarized protons via their passage through a polarized hydrogen storage cell [20]. Hadronic and electromagnetic interactions induce differing losses for distinct orientations of the spin of a proton [21].

Proton proton spin observables are known at low energy as a result of many measurements and through the use of phase shift analyses. Spin dependent hadronic cross sections at low momenta provide a contribution that facilitates the rate of polarization buildup in the spin filtering experiment [22]. Electromagnetic effects refine the discussion of an explanation of the rate of increase of proton polarization as the cross sections are well understood [23].

The spin dependent cross sections for elastic antiproton proton scattering are not well known [24]. Models of the antiproton proton interaction have appeared [25] that suggest a similar arrangement may be useful in seeking to polarize an antiproton beam [26].

To investigate the evolution in time t of polarization for particles circulating in a storage ring at frequency v through a polarized target of areal density n and polarization P oriented normal to the ring plane, consider the following coupled system [27]

$$\frac{d}{dt}\begin{bmatrix}N\\J\end{bmatrix} = -nv\begin{bmatrix}a&b\\c&d\end{bmatrix}\begin{bmatrix}N\\J\end{bmatrix}$$
(12)

describing the rate of change of the number of beam particles N(t) and their total spin J(t), the difference between the number of particles polarized in a particular direction of interest and the number polarized in the opposite direction [28]. The quantity N(t) relates to the sum of the polarized species. Loss of particles from the beam involves integration beyond a ring acceptance angle, θ_{acc}

$$a = 2\pi \int_{\theta_{\rm acc}}^{\pi} \frac{d\,\sigma}{d\,\Omega} \,\sin\theta\,d\theta\,.$$
(13)

Beam loss is induced by scattering over all angles beyond θ_{acc} , the acceptance angle of the storage ring [29].

Polarized leptons

Antiproton polarization may be transferred from polarized leptons also [30]. The spin averaged differential cross section appearing in the element *a* of the above evolution matrix for the elastic scattering of antiprotons of mass m_p on electrons of mass m_e is, to leading order in inverse momentum transfer $1/q^2$,

$$s\frac{d\sigma}{d\Omega} = \left(\frac{2\,\alpha\,m_e E_l}{q^2}\right)^2 \tag{14}$$

where E_i is the laboratory energy of the incident hadron. Changes in the total spin J due to matrix element c involve spin transfer K_{ii} and asymmetry A_{ii} where $i, j \in \{L, N, S\}$. Such parameters are integrated over particular angular ranges and multiplied by P, the polarization of the target. Depolarization relates to the final element d of the above rate matrix. Spin transfer and asymmetry parameters for polarization normal to the scattering plane have singular $1/q^2$ terms

$$s\frac{d\sigma}{d\Omega}K_{NN} = s\frac{d\sigma}{d\Omega}A_{NN} \approx \frac{2\alpha^2}{q^2}\mu_p m_e m_p \tag{15}$$

where μ_p is the magnetic moment of the hadron in units of a nuclear magneton, and where, in the expression on the right hand side, we have written $\mu_e = 1$ for the magnetic moment of the electron. The longitudinal spin transfer and asymmetry parameters also display singular $1/q^2$ terms near the forward direction

$$s\frac{d\sigma}{d\Omega}K_{LL} = s\frac{d\sigma}{d\Omega}A_{LL} \approx \frac{2\alpha^2}{q^2}\mu_p m_e E_l.$$
(16)

The electromagnetic spin transfer observables in the scattering plane, $K_{SS} = A_{SS}$, are not singular in q and make little contribution. The same applies to the depolarization parameter $1 - D_{NN}$ which likewise is not singular in q and may be ignored. Some baryon depolarization, though, does occur due to the singular terms

$$(1 - D_{LL})s\frac{d\sigma}{d\Omega} = (1 - D_{SS})s\frac{d\sigma}{d\Omega} \approx \frac{2\alpha^2}{q^2} \left(\frac{m_p m_e E_l}{k}\right)^2$$
(17)

where k refers to the magnitude of the centre of mass three momentum. A change in the total spin J results from the matrix element c which has a contribution involving a product of P with the spin transfer observable integrated over angles θ above a minimum angle θ_0 linked to the average distance between charges, an impact parameter beyond which scattering is inhibited. For target polarization normal to the storage ring, and for scattering at random azimuthal angles, integration over such angles requires consideration of the polar integration of the two-spin observables [31]

$$P \pi \int_{\theta_0}^{\theta_{\rm acc}} \left(K_{\rm NN} + K_{\rm SS} \right) \frac{d \sigma}{d \Omega} \sin \theta \, d\theta \tag{18}$$

integrated over angles such that particles remain in the ring. In contrast to the transverse case, where an azimuthal average leads to the appearance of both asymmetries $K_{\rm NN}$ and $K_{\rm SS}$, the case of longitudinal target polarization merely requires the following polar integral

$$P \pi \int_{\theta_0}^{\theta_{\rm acc}} 2K_{\rm LL} \, \frac{d\,\sigma}{d\,\Omega} \,\sin\theta \, d\theta \,. \tag{19}$$

A contribution to a change in J involving J itself, part of the matrix element d of the rate matrix, results from a similar integral over the azimuthally averaged depolarization observable, below acceptance. In the short term, the rate of increase of hadron polarization J/N is given approximately by

$$\frac{dJ}{dt} \approx -n v c N \tag{20}$$

where c relates to integrals over double spin asymmetries that have singular behaviour in q and are consequently enhanced by logarithmic factors involving, for example, the areal density n

$$\ln(q_{\rm acc}^2/q_0^2) = \ln(q_{\rm acc}^2/n).$$
(21)

Transfer at Low Energy

In a distorted wave approximation involving Coulomb effects, a group from Mainz finds in a study, part analytical and part numerical, that the angle integrated polarization transfer cross section for proton electron or antiproton positron elastic scattering, namely [32]

$$\left|\left\langle A_{\rm LL}\frac{d\sigma}{d\Omega}\right\rangle\right|$$

is many orders of magnitude greater than that expected from a plane wave one photon exchange calculation over a range of incident proton laboratory kinetic energies from 1 MeV down to 1 keV at which low energy the distorted wave approximation is no longer valid as an approach to a bound state singularity beckons [33]. The same would apply for antiproton positron collisions. A Budker group has provided an analytical calculation of the same cross section and has indicated that, even so, the time taken to polarize a hadronic fermion is still very long with currently available lepton densities [34].

It would be interesting to know if the double spin asymmetry and depolarization observables show a similar dramatic change with reducing incoming energies. Studies incorporating cooling mechanisms may be concerned with the value at low energies of the integrated depolarization cross section over angle, though it appears that its contribution would be negligible. The large increase of the double spin asymmetry with energy as incident momenta decrease to about 1 keV, a lower limit for the distorted wave approximation, compensates to some extent for the low lepton areal density when comparison is made with a method using storage cells.

CONCLUSIONS

It is clear that the availability of a polarized antiproton beam would probe the detailed spin dependence of antimatter collisions and would facilitate unprecedented tests of QCD transversity. A number of methods have been suggested in seeking to achieve polarization, among them the technique of channelling. Another method considers the use of spin transfer from polarized atoms and their constituents, and a third, the notion of transferring polarization from a beam of leptons, electrons or positrons, or, conceivably, muons.

The efficiency with which antiprotons enter a well prepared crystal is a concern when evaluating a method of channelling that endeavours to induce polarization by encouraging passage through a curved lattice of nuclei, necessarily of considerable length. Moreover, the size of the analyzing power deep within the electromagnetic hadronic interference region is limited by the requirement that the crystal can only be bent to a certain degree, not to mention the risk of failing to keep the projectile close to an attracting row of lattice nuclei due to an uncontrolled centrifugal force resulting from too enthusiastic a curvature.

The use of the spin dependence of hadronic and electromagnetic cross sections in selectively attenuating an antiproton beam as it collides with a polarized hydrogen target offers the most persuasive technique for achieving polarization. Though the spin observables need more refined measurement for antiproton proton elastic scattering, the hope is that the success of the proton proton programme will serve as a guide as to what may be forthcoming in the antiparticle case.

Transferring spin from a polarized beam of leptons has the attractive feature that the appropriate cross sections may be calculated with confidence, in principle, by using our detailed understanding of quantum electrodynamics. The subtleties of the evaluation at lower energy, however, are being drawn to the surface due to the pressure of attempting to achieve an estimate of the overall effect. The rate of transfer of polarization does seem to be in need of enhancement. Nevertheless, nothing should be left undone on the margin

of the impossible particularly when more intense electron or positron beams may appear as a result of requirements in another sphere.

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